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Article

Modelos para estimar el volumen comercial de Bursera simaruba (L.) *Sarg. y de Metopium brownei (Jacq.)* Urb. en Quintana Roo, México

Models to determine the commercial volume of Bursera simaruba (L.) *Sarg. and of Metopium brownei (Jacq.)* Urb. in Quintana Roo, México

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Resumen

El estado de Quintana Roo ocupa el segundo lugar nacional en producción maderable de especies latifoliadas; de ellas, Bursera simaruba y Metopium brownei son de relevancia forestal debido al valor de su madera y amplia distribución en la entidad. Para elaborar programas de manejo forestal, las funciones que estiman el volumen fustal y comercial, además del ahusamiento y la razón de volumen son importantes. El objetivo del presente estudio fue ajustar una función no-lineal que estime el volumen comercial para dos taxa en el centro y sur de Quintana Roo, México. Con información dasométrica de 188 y 133 árboles, respectivamente que abarcaron todas las categorías diamétricas observadas en estos ecosistemas forestales; se ajustó un modelo de volumen comercial con efectos aleatorios a nivel de árbol, y se eligieron las mejores combinaciones de inclusión de los efectos aleatorios según la máxima verosimilitud. Las ecuaciones propuestas tienen un sesgo, en promedio, al estimar el volumen comercial para los dos taxones de 0.0045 m3 y una explicación mayor a 90 % de la variabilidad muestral. Las ecuaciones resultantes podrán emplearse en la estimación maderable en los programas de manejo forestal sustentable de los bosques tropicales ubicados en el centro y sur del estado.

Palabras clave: Bosque tropical, distribución de productos, manejo forestal, sistema de cubicación, Quintana Roo, volumen.

Abstract

The state of Quintana Roo occupies the second national place in timber production of broadleaf species, of which Bursera simaruba and Metopium brownei are relevant, from the value of their wood and wide distribution. The functions that estimate the stem and commercial volume, in addition to the taper and the volume ratio are important in the elaboration of a forest management program. The aim of this study was to fit a nonlinear function that estimates trade volume for two species in central and southern Quintana Roo. A commercial volume model with random effects was fitted at the tree level, using the mensuration data from 188 and 133 trees for each species, respectively, and all the diameter categories found in these forest ecosystems were covered. The best combinations of inclusion of the random effects were chosen according to the maximum likelihood. The proposed equations have an average bias when estimating the commercial volume for the two species of 0.0045 m3 and an explanation greater than 90 % of the sample variability. The resulting equations may be used in the timber estimation of the sustainable forest management programs of tropical forests at the central and southern territories of the state.

Key words: Tropical forest, product distribution, forest management, cubing system, Quintana Roo, volume.

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Introduction

The state of *Quintana Roo* (*Q. Roo*) occupies the second national place in timber production of broad-leaved species, with an approximate harvest of 47 139 m^3 of roundwood (m^3 mr) of common tropical species (12.10 % of national production) and of 4 807 m^3 mr of precious species (15.18 % of national production), with a value of about $ 84 091 064 and $ 17 306 518, respectively, which places it with a contribution to GDP below 0.2 % registered at national level, according to the Ministry of Environment and Natural Resources (Semarnat, 2017). *Bursera simaruba* (L.) Sarg. (*Chakáh*) and *Metopium brownei* (Jacq.) Urb. (*Chechem negro*) are common tropical species of commercial interest for the forestry industry (Silva *et al.,* 2011; Gallegos *et al.,* 2012), they are widely distributed in the Peninsula of *Yucatán* (Herbario CICY, 2010), are used for agroforestry for their growth and regeneration (*Bursera simaruba*) as well as for the production of furniture and handicrafts (*Metopium brownei*) (Román *et al*., 2016; ITTO, 2020).

The accurate estimation of the stem volume, of the total volume when including the branches, and of the commercial volume based on the dimensions that the market demands, is important to develop the silvicultural practices proposed in the forest management plan according to the data of the forest inventories carried out (Corral-Rivas and Návar-Cháidez, 2009; Diéguez-Aranda *et al.,* 2009; Gómez-García *et al.,* 2016; Tamarit *et al.,* 2017; Niño *et al.,* 2018).

Mathematical modeling is a support tool for forest resource managers, for which the adjustment of models that estimate the stem, total and commercial volume; o model, with precision, the ratio between the commercial volume and the stem *(Vc / Vf)*, as well as its dynamics of decrease in the accumulated volume of a tree with the increase in height and the reduction of the diameter on the stem until reaching to the total height (volume ratio functions), in addition to describing the taper of the trees (diameter at different heights) by species, contributes to making management plans according to the specific and current conditions of the resources (Burkhart, 1977; Tapia and Návar, 2011; Barrios *et al*., 2014; Özçelik and Göçeri, 2015; Hernández-Ramos *et al*., 2018a).

In recent years in Mexico, modeling has been carried out through functions that consider diameters at different heights on the stem and compatible commercial taper-volume systems for different species of the *Pinus* genus (Tapia and Návar, 2011; Quiñonez-Barraza *et al*., 2014; Tamarit *et al*., 2016; Silva *et al*., 2018), or by volume ratio models for species such as *Eucalyptus urophylla* Blake in *Huimanguillo, Tabasco, Swietenia macrophylla* King in *Quintana Roo* and *Pinus pseudostrobus* Lindl. in *San Juan Nuevo Michoacán*, among others (Hernández-Ramos *et al*., 2018a; García-Espinoza *et al*. 2018).

Commercial volume systems composed of ratio models are a tool to accurately calculate the commercial volume and the distribution of timber products from standing trees (Prodan *et al*., 1997; Hernández-Ramos *et al*., 2018a; García-Espinoza *et al.,* 2018). The estimation of these volumes is fundamental in the projection of the use of the timber resource and the yield for the forest industry, because they make it possible to estimate the financial income of the forest from forest inventory data, mainly the distribution of products per unit area. (Niño *et al.,* 2018; Silva *et al.,* 2018); and they also contribute to avoiding the overuse of species of commercial importance in general, not only of tropical forests.

For the area of interest, Hernández-Ramos *et al.* (2018a and 2018b) adjusted total volume models and generated a commercial volume system from ratio models, in addition to a segmented taper system *(d)* compatible with commercial volume (*Vc*) for *Swietenia macrophylla* King (mahogany) in tropical forests of *Quintana Roo*. López *et al.* (2019) adjusted a compatible system of commercial volume equations consisting of stem, branches and stem and thick branches of trees for 11 important tropical species in southeastern Mexico.

Traditionally, the equations to design these commercial volume systems have been adjusted using linear and non-linear regression models that assume normality, equality of variances and independence of errors (Carrero *et al*., 2008).

However, obtaining reliable equations for some variables is complex, due to the variability that exists in these relationships due to the effect of the different forms of the stems and the conditions in which each individual develops (Quiñonez *et al.,* 2012). At present, the adjustment under the approach of mixed effects models (MEM) has become a viable option for this type of study (Corral-Rivas *et al*. 2014), since with its incorporation in its parameters it allows grouping the sources of variability and reduce the variance of the error (Seoane, 2014). This MEM technique includes in its formulation fixed parameters common to the entire population and specific random parameters for each level of grouping (Seoane, 2014). As a result, more efficient, precise and reliable estimators result than the fixed parameters of the model and, therefore, estimates can be made with greater certainty of the variable of interest (Carrero *et al.,* 2008). For all the above, the objective of the present study was to fit a non-linear function with random effects that estimates the commercial volume for *Bursera simaruba* and *Metopium brownei* in central and southern *Quintana Roo*, Mexico.

Materials and Methods

The study was carried out in different forest areas of *Quintana Roo*, Mexico, which are located in areas with topoforms of low hills and rocky plains at an altitude of 20-180 m. The predominant climate is warm subhumid (Aw), with an average annual temperature of 22 °C and an average rainfall of 1 300 mm (Semarnat, 2014).

From tours in the forest areas of the state, trees larger than 7.5 cm in normal diameter were selected randomly located in undisturbed areas of the tropical forest, where 188 specimens of *Bursera simaruba* and 133 of *Metopium brownei* were sampled, with which they were covered all the diameter categories found in these forest ecosystems (10—50 and 10—60 cm, respectively). Information on stump diameter *(dt*), normal diameter (*d*), total height (*H*), and partial diameters and heights on the stem (d_i and h_i) were taken from each of them. The sample was chosen with monopod trees, without the presence of mechanical damage or pests or diseases, in addition to being at least 20 m away from roads, clearings or places with clearings to avoid edge effects. The descriptive statistics of the analyzed variables are arranged in Table 1.

Table 1. Descriptive statistics of the sample of *Bursera simaruba* (L.) Sarg. and Metopium brownei trees (Jacq.) Urb.

Statistics	**Species**									
	Bursera simaruba					***Metopium brownei***				
	d	***H***	***Vt***	***R***	***Vc***	***d***	***H***	***Vt***	***R***	***Vc***
Average	23.82	13.97	0.4938	0.44	0.2143	30.00	15.45	0.6820	0.46	0.3174
Typical error	0.24	0.08	0.0110	0.01	0.0080	0.30	0.09	0.0140	0.01	0.0120
Standard deviation	9.21	3.19	0.4360	0.38	0.3200	10.13	3.11	0.4840	0.37	0.3950
Coefficient of variation	38.66	22.88	88.2900	87.05	149.2900	33.77	20.13	70.9600	81.52	124.4300
Variance	84.79	10.22	0.1900	0.14	0.1020	102.64	9.67	0.2350	0.14	0.1560
Curtosis	-0.50	-0.60	1.5000	-1.50	2.3000	-0.60	0.50	-0.1000	-1.60	3.2000
Coefficient of asymmetry	0.60	-0.10	1.4000	0.50	2.6000	0.20	-0.50	0.8000	0.40	1.9000
Minimum	7.00	5.57	0.0239	0.01	0.0013	9.40	4.63	0.0337	0.02	0.0028
Maximum	49.80	21.80	2.1000	1.00	2.1000	57.30	23.10	2.3000	1.00	2.3000

d = Normal diameter (cm); *H* = Total height (m); *Vt* = Total volume (m^3); *R* = Volume ratio (m^3); *Vc* = Commercial volume (m^3).

The measurements of the diameters (*di*) at different heights on the stem (h_i) of 0.3 m, 0.6 m, 0.9 m, 1.3 m and 2.5 m, were made directly with a caliper graduated to the millimeter, while the rest of h_i and d_i with a Bitterlich Telerelascope along the shaft where possible from 2.5 m to full height (*H*) or zero diameter (d_0). For the calculation of the stem volume (*Vf*) the volumes of the different sections (logs) were added, which were determined by the Smalian formula and for the volume of the tip or last log (*Vp*) with the cone formula; the volume ratio (*R*) resulted from the ratio between the volume at different sections on the stem (*Vi*) and the *Vf,* for which the expression *Vi/ Vf* was used (Balzarini *et al*., 2005; Hernández-Ramos *et al*. , 2018a; García-Espinoza *et al.* 2018).

As indicated in the Official Mexican Standard NOM-152-SEMARNAT-2006, a non-linear function (general model) for each species was adjusted, which is made up of two components: the first one includes a mathematical structure that corresponds to an equation of *Vf* ($a_0+a_1(d^2H/1000)$ and the second one is an equation referring to the ratio between the volumes of the sections and the *R* ($e^{(-(a2)*((di/d)/1000)}e^{(a3*(di/d))}$), while these equations, when multiplied, estimate the commercial volume (*Vc*) (Schabenberger and Pierce, 2002; Balzarini *et al*., 2005) (Table 2).The latter is defined by the dimensions that the sawmill industry or timber products demand, in which there is an interest in estimating, whether long dimensions or primary, short or secondary and waste or firewood. For the two species, the general model and the random effect additive to each combination of the fixed parameters were tested and eight expressions were obtained from the general model.

Table 2. Adjusted commercial volume models with fixed and random effects for *Bursera simaruba* (L.) Sarg. and *Metopium brownie* (Jacq.) Urb.

Identifier	Structure of the models
General Identifier	$Vc = \left[a_0 + a_1 \cdot \left(d^2 \cdot H/_{1000}\right)\right] \cdot \left[e^{(-a_2 \cdot (\frac{d_i}{d}/1000))} \cdot e^{a_3 \cdot \frac{d_i}{d}}\right]$
1	$Vc = \left[(a_0 + u) + a_1 \cdot \left(d^2 \cdot H/_{1000}\right)\right] \cdot \left[e^{((-a_2+u) \cdot (\frac{d_i}{d}/1000))} \cdot e^{a_3 \cdot \frac{d_i}{d}}\right]$
2	$Vc = \left[(a_0 + u) + a_1 \cdot \left(d^2 \cdot H/_{1000}\right)\right] \cdot \left[e^{(-a_2 \cdot (\frac{d_i}{d}/1000))} \cdot e^{(a_3+u) \cdot \frac{d_i}{d}}\right]$
3	$Vc = \left[a_0 + (a_1 + u) \cdot \left(d^2 \cdot H/_{1000}\right)\right] \cdot \left[e^{((-a_2+u) \cdot (\frac{d_i}{d}/1000))} \cdot e^{a_3 \cdot \frac{d_i}{d}}\right]$
4	$Vc = \left[a_0 + (a_1 + u) \cdot \left(d^2 \cdot H/_{1000}\right)\right] \cdot \left[e^{(-a_2 \cdot (\frac{d_i}{d}/1000))} \cdot e^{(a_3+u) \cdot \frac{d_i}{d}}\right]$
5	$Vc = \left[(a_0 + u) \cdot \left(d^2 \cdot H/_{1000}\right)^{a_1}\right] \cdot \left[e^{((-a_2+u) \cdot (\frac{d_i}{d}/1000))} \cdot e^{a_3 \cdot \frac{d_i}{d}}\right]$
6	$Vc = \left[(a_0 + u) \cdot \left(d^2 \cdot H/_{1000}\right)^{a_1}\right] \cdot \left[e^{(-a_2 \cdot (\frac{d_i}{d}/1000))} \cdot e^{(a_3+u) \cdot \frac{d_i}{d}}\right]$
7	$Vc = \left[a_0 \cdot \left(d^2 \cdot H/_{1000}\right)^{(a_1+u)}\right] \cdot \left[e^{((-a_2+u) \cdot (\frac{d_i}{d}/1000))} \cdot e^{a_3 \cdot \frac{d_i}{d}}\right]$
8	$Vc = \left[a_0 \cdot \left(d^2 \cdot H/_{1000}\right)^{(a_1+u)}\right] \cdot \left[e^{(-a_2 \cdot (\frac{d_i}{d}/1000))} \cdot e^{(a_3+u) \cdot \frac{d_i}{d}}\right]$

d = Normal diameter (cm); d_i = Diameter at different heights on the stem; H = Total height (m); Vc = Commercial volume; a_0, a_1, a_2 and a_3 = Parameters to be estimated; $+u$ = Random effect; e = Exponential function.

The general model (Table 2) was first adjusted by ordinary least squares (OLS) and two variants of Spurr's (1952) shaft volume equation: a linear one of the $y = a_0 + a_1 X$ form, and the second one of the exponential type $y = aX^b$ form. In them, y is the response variable (Vf = stem volume), X represents the independent variable included in the model (d^2H), a and b correspond to the parameters to be estimated.

The incorporation of the random effect to the fixed parameters ($a_i + u$) was done per tree to explain the variability in its tapers, considering the biological meaning of each one; for example, by including the random effect in the parameter referring to the shape of the tree, the rate of change of the geometric body in the stem, the ratio of proportion between the stem volume and the commercial volume or in all three simultaneously within each of the components in the proposed non-linear function, where the probability value of the effect must be significant ($P \geq 0.05$) (Table 1, 1-16).

The adjustment was carried out in the SAS 9.3® program using the nlmixed procedure and the FIRO first order method, which defines with greater precision the approximation by likelihood of the values in the estimated parameters (Beal and Sheiner, 1982; SAS, 2011) The distribution for both the dependent variables and the random effects ($a_i + u$) was specified in the programming as NORMAL to be adjusted by the Dual Quasi-Newton optimization technique and the Adaptive Gaussian Square integration method of the Statistical Analysis System (SAS, 2011).

The selection of the best combination when including the random effects within the non-linear model, was made through the significance of the estimated parameters in both fixed and random effects at a value of 95 % reliability ($P \geq 0.05$), and the best values of Verisimilitude-2Log and Information Criterion of Corrected Akaike (*CIAC*, for its acronym in Spanish) (García and Rapelli, 2011; Gómez-García *et al*., 2016). Furthermore, for the best expression, the trend in the estimates was graphically verified by using the model structures separately in the stem volume (*Vf* = first mathematical structure) and volume ratio (*R* = second mathematical structure), and so joint to estimate the commercial volume (*Vc*) of each species or up to a diameter of 20 cm (Table 1, general model).

In the selected model, the homoscedasticity of the residuals was verified using White's test, under the hypothesis that the residuals are homogeneously distributed around zero (null hypothesis: *Ho*), while the alternative hypothesis refers to a homoscedastic distribution of the residuals (alternative hypothesis: *Ha*) at a 95 % reliability level (Martínez *et al*., 2006). The quality of the estimates for the variables *Vf*, *R* and *Vc* was determined through bias (*E*) (Quevedo *et al*., 2010) and the coefficient of determination (R^2) and through the graphic analysis of the observed values against the values estimates of each variable (Pece, 1994).

Results

When adjusting the general model and the variations in the inclusion of random effects, it was observed that both the general structure of the model and expressions 1, 2, 4 and 5 for *Bursera simaruba* and 1, 2, 4 and 5 for *Metopium brownei*, in some of its fixed parameters, they were not significant at 95 % reliability ($P \geq 0.05$). While, in the combinations of inclusion of the random effects of expressions 6 and 8 in both species, the parameters related to mixed affects could not be estimated even when they tried to adjust by the Gauss-Hermite method (Pinheiro and Bates, 1995). Therefore, these models were discarded for further analysis and were not included in Table 3. Due to this situation, only the combinations of inclusion of the random parameters of expressions 3 and 7 for *Bursera simaruba* and *Metopium brownei* trees are presented for subsequent analyzes, in which all were significant ($P \geq 0.05$) when adjusted with 1 476 and 1 130 pairs of data, respectively.

Table 3. Estimated parameters for the different expressions of the adjusted commercial volume model for *Bursera simaruba* (L.) Sarg. and *Metopium brownie* (Jacq.) Urb.

Species	System	Parameter	Estimator	*Eea*	t Value	Pr>|t|	ICI	ICS
Bursera simaruba	3	a_0	0.01238	0.0059	2.09	0.0381	0.00068	0.02407
		a_1	0.04866	0.0011	42.69	<0.0001	0.04642	0.05091
		a_2	9.15250	0.9572	9.56	<0.0001	7.26420	11.04070
		a_3	5.32220	0.1084	49.09	<0.0001	5.10840	5.53610
		se	-0.04927	0.0009	-50.28	<0.0001	-0.05121	-0.04734
		su1	0.00861	0.0006	13.71	<0.0001	0.00737	0.00985
		su2	0.91109	0.1575	5.79	<0.0001	0.60120	1.22250
	7	a_0	0.05710	0.0024	23.37	<0.0001	0.05228	0.06192
		a_1	0.94400	0.0200	47.09	<0.000 1	0.90440	0.98350
		a_2	9.18550	0.9675	9.49	<0.0001	7.27700	11.09400
		a_3	5.32100	0.1092	48.73	<0.0001	5.10560	5.53640
		se	-0.04964	0.0009	-50.26	<0.0001	-0.05158	-0.04769
		su1	-0.07213	0.0062	-11.45	<0.0001	-0.08456	-0.05971
		su2	-0.93620	0.1633	-5.73	<0.0001	-1.25830	-0.61400
Metopium brownei	3	a_0	0.00807	0.0118	0.68	0.4977	-0.01541	0.03155
		a_1	0.04111	0.0013	29.79	<0.0001	0.03838	0.04384
		a_2	3.08900	0.5345	5.78	<0.0001	2.03160	4.14640
		a_3	6.30480	0.1792	35.18	<0.0001	5.95030	6.65930
		se	-0.07122	0.0016	-43.68	<0.0001	-0.07444	-0.06799
		su1	-0.00954	0.0007	-12.65	<0.0001	-0.01103	-0.00805
		su2	0.36620	0.0785	4.66	<0.0001	0.21090	0.52150
	7	a_0	0.04722	0.0040	11.57	<0.0001	0.03914	0.05529
		a_1	0.95240	0.0333	28.58	<0.0001	0.88650	1.01830
		a_2	3.09670	0.5375	5.76	<0.0001	2.03330	4.16000
		a_3	6.30360	0.1798	35.06	<0.0001	5.94800	6.65930
		se	-0.07144	0.0016	-43.52	<0.0001	-0.07468	-0.06819
		su1	-0.08848	0.0079	-11.1	<0.0001	-0.10430	-0.07271
		su2	0.37550	0.0810	4.63	<0.0001	0.21520	0.53580

a_0, a_1, a_2 and a_3 = Fixed effect parameters; su_1 and su_2 = Standard deviation of random effects; *Eea* = Estimator standard error; se= Standard deviation of the residuals in the system; ICI and ICS = Confidence intervals of the lower and upper estimates, respectively.

For the two species, the combination in the mathematical structure that contains the Spurr stem volume model in its exponential form, presents the highest statistical values in the likelihood indicators -2Log and CIAC, for which the structures of model 7 were selected for *Bursera simaruba* and *Metopium brownei*, respectively. When contrasting these values with those obtained when adjusting the general model by ordinary least squares (OLS), an average statistical gain is observed when applying the mixed effects models approach of 5.6 % and 6.9 % for *Bursera simaruba* and *Metopium brownei*, respectively (Table 4). In addition, White's tests for both species were not significant (a < 0.05), which indicates that the residuals of both models are distributed in a homoscedastic way, which is why the null hypothesis (*Ho*) is accepted and *Ha* is rejected.

Table 4. Statistical fit indicators of the adjusted trade volume model for *Bursera simaruba* (L.) Sarg. and *Metopium brownie* (Jacq.) Urb.

Species	***Bursera simaruba***			***Metopium brownei***		
Random effects model	**General model**	**3**	**7**	**General model**	**11**	**15**
Verisimilitude -2 *Log*	-4 617	-4 368	-4 360	-2 588	-2 409	-2 396
CIAC	-4 601	-4 354	-4 346	-2 543	-2 395	-2 382

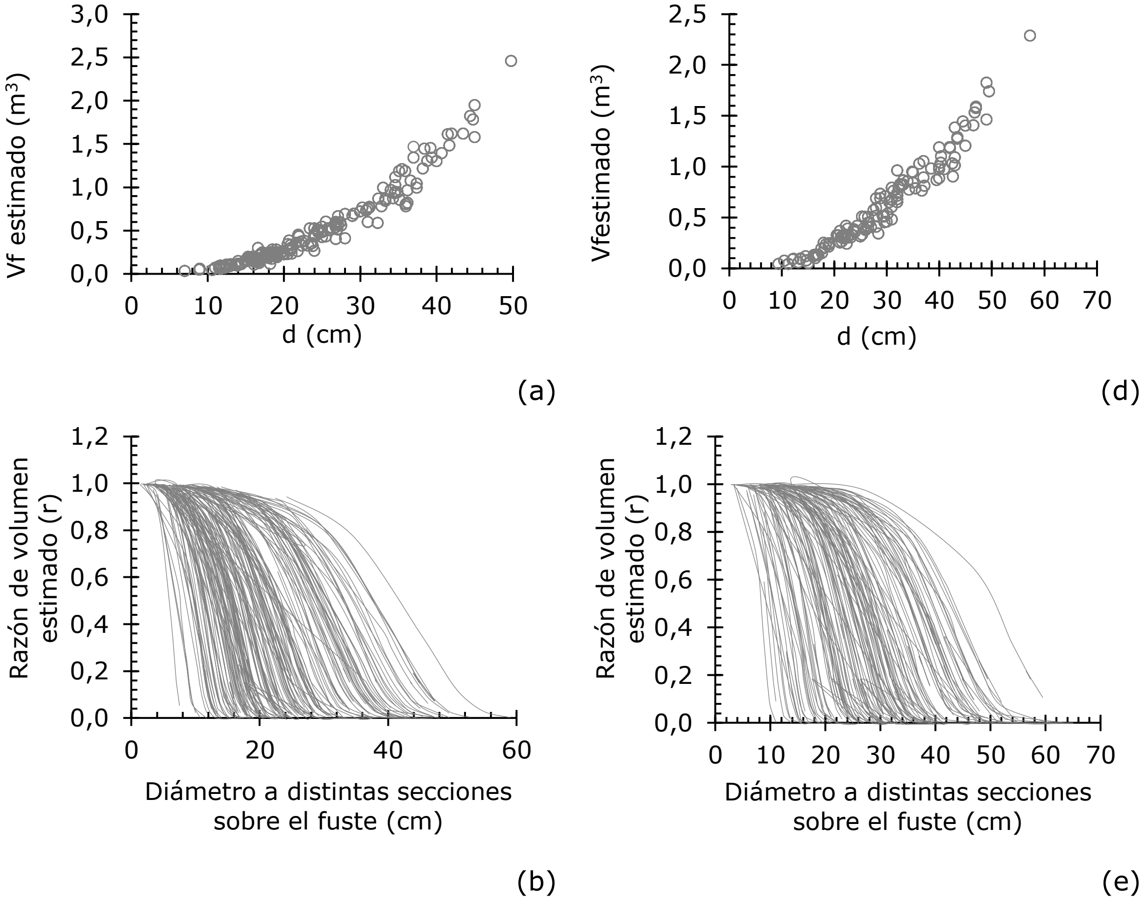
CIAC = Information Criterion of Corrected Akaike.

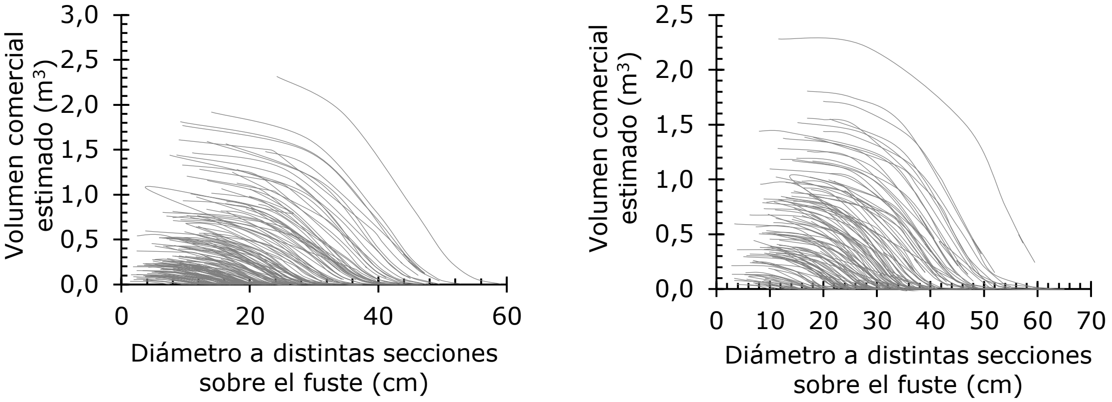
The biases estimated for the two species show good precision in the estimates when evaluating them individually for *Vf* and *R*, and as a complete model to estimate *Vc*, since in all cases they are less than 0.03 m^3 (Table 5). In addition, when performing the graphical evaluation and by means of a regression of the observed values against those predicted by the variable of interest, no obvious problems were observed in the estimates, a situation that is reflected in the value of the determination coefficient (R^2) that in all cases was greater than 0.9 (Figure 1).

Table 5. Estimated bias by independent variable in the adjusted commercial volume model.

Independiente variable	**Vf (m^3)**	**R (V_i/Vf)**	**Vc (m^3)**
Model including the random effects		**Sesgo (E)**	
7. *Bursera simaruba* (L.) Sarg.	-0.012543 ($R^2 = 0.95$)	0.029831 ($R^2 = 0.93$)	-0.001384 ($R^2 = 0.94$)
7. *Metopium brownei* (Jacq.) Urb.	0.014684 (R^2 = 0.93)	0.019924 ($R^2 = 0.94$)	0.008979 ($R^2 = 0.92$)

Vf = Stem volume; R = Volume ratio based on diameter; Vi = Volume at different sections on the shaft; Vc = Commercial volume; R^2 = coefficient of determination of the observed values against the predicted values.





(c) (f)

Figure 1. Trend of stem volume estimates, volume ratio and commercial volume for *Bursera simaruba* (L.) Sarg. (1a, 1b and 1c) and *Metopium brownei* (Jacq.) Urb. (1d, 1e and 1f) species.

To exemplify the use of the structure that best fit the observed data of the commercial volume model, the values of normal diameter *(d)* = 30 cm and total height *(H)* = 15 m for *Bursera simaruba,* and *d* = 40 cm and *H* = 20 m for *Metopium brownei,* using model 7 with its parameters for each species, respectively, to estimate the stem volume *(Vf)* and volume ratio *(R)* when using the two sections of the model separately and when applying the complete model adjusted for commercial volume *(Vc).* From the general expression of the model, the first component corresponding to *Vf* is taken, which, when substituting for *Bursera simaruba* there is:

$Vf = 0.0571\ ((30^{2} \cdot 15)/1000)^{0.9440} = 0.6663\ \text{m}^{3}$

And for *Metopium brownei:* $Vf = 0.0472\ ((40^{2} \cdot 20)/1000)^{0.9524} = 1.5049\ \text{m}^{3}$.

Next, the minimum diameter for primary products is established, which in this case was 20 cm; then the second section of the general model is taken and the values are substituted, for which the following results for *Bursera simaruba*:

$R = Exp(-9.1855\cdot((20/30)/1000)\cdot Exp(5.321*(20/30)))=$ 0.8085, y para *Metopium brownei:* $R = Exp(-3.0967\cdot((20/40)/1000)\cdot Exp(6.3067*(20/30)))$ = 0.9644.

Finally, to obtain the *Vc* at the established minimum cutting diameter, the values of *Vf* and *R* are multiplied, or the proposed model of each species is applied directly,: for *Bursera simaruba* as follows:

$Vc = [0.0571\ ((30^{2}\cdot 15)/1000)^{0.9440}]\cdot[Exp(-9.1855\cdot((20/30)/1000)\cdot Exp(5.321*(20/30)))] = 0.5387\ m^{3}$, y para *M. brownei:*

$Vc = [0.0472((40^{2}\cdot 20)/1000)^{0.9524}]\ [Exp(-3.0967\cdot((20/40)/1000)\cdot Exp(6.3067*(20/30)))] = 1.4514\ m^{3}$.

Discussion

The MEM allows correcting the structure of variances-covariances associated with the classification variables of the trees (Littell *et al*., 2006), since by grouping the information in class, under the MEM approach, the error is reduced, since the variability between the classes is explained by the inclusion of random parameters, which are estimated simultaneously with the fixed parameters (Castedo *et al*., 2006; Corral *et al*. 2019). In this analysis technique (MEM), it assumes that the variance is specific for each classification level (Seoane, 2014), so it is possible to increase the R^2 and improve other goodness-of-fit criteria, such as the AIC and BIC.

In the evaluated models, random effects were included to account for the effect of classification covariates during adjustment in the equations. In this case, the random effect was specific at each level of the classification factor, which according to De los Santos *et al.* (2006), Tamarit-Urias *et al.* (2014) and García-Espinoza *et al.* (2019) allows obtaining more efficient, precise and reliable estimators of the fixed parameters of the model and predicting specific random parameters of each sample unit, which reflect the deviation pattern with respect to the mean, but after adjustment, the values obtained from the fit for the random parameters are not used to construct the predictions, so the parsimony of the models is not modified.

In the case of the structure of the volume model 3 for *Bursera simaruba*, and 7 for *M. brownei*, all the parameters were significant (P ≥ 0.05) (Table 3), similar to that reported by Quevedo *et al.* (2010), these authors analyzed the stem volume models under the mixed effects approach for *Tabebuia rosea* (Bertol.) In *Venezuela*, where a grouping by diameter category and transformation of the dependent variable was performed. In a similar way to the study carried out by Gómez-García *et al*. (2016), who developed a profile function through mixed effects for *Pinus sylvestris* L. in Turkey, and where it was adjusted by the FIRO (first-order) method, of the R-project program, which is the same method used in this work.

For the choice of model 3 and 7 of Table 1 for *Bursera simaruba* and *Metopium brownei*, respectively, the description by García and Rapelli (2011) was taken as a reference, who reported that the models with the values closest to zero in the indicators of Verisimilitude -2Log and CIAC have a higher likelihood in the fit. In addition, these values were consistent with the evaluation criteria made by Gómez-García *et al.* (2016); and by Quiñonez-Barraza *et al*. (2014), who adjusted compatible commercial taper-volume systems for the main *Pinus* species from *Durango*, Mexico.

The bias values in the models proposed by species and the determination coefficients (R^2) obtained by contrasting the observed data against those predicted by the variable of interest (*Vf, R* and *Vc*), coincide with that reported by Pece (1994), who used a volume ratio model to estimate the commercial volume in *Eucalyptus pellita* F. Muell. in Brazil and obtain biases when calculated by diameter category between 0.06 and 0.11. Furthermore, it is similar to that determined by Hernández-Ramos *et al*. (2017), by proposing volume ratio systems for *Eucalyptus urophylla* in *Tabasco* and obtaining accurate estimates with lower biases in the variables of *Vt, R* and *Vc* than those obtained in this work (< 0.0077) and R^2 when contrasting the estimated values *versus* observed greater than 0.94.

Likewise, the precision of the estimates obtained by this work in commercial volume (*Vc*) are similar to that recorded by Hernández-Ramos *et al.* (2018 b) when using volume ratio systems in *Swietenia macrophylla* trees in *Quintana Roo* and obtaining biases for volume ratio structures based on normal diameter and total height of -0.0121 and 0.0053, respectively. However, they differ with García-Espinoza *et al.* (2018) by proposing structures based on the volume ratio for *P. pseudostrobus* in *Michoacán*, and registering good accuracy in the estimates and a lower bias in the *Vc* compared to those obtained in this work (< 0.001 m^3).

The graphical evaluation of the trends in the estimates (Figure 1) agrees with that obtained by Tapia and Návar (2011) who adjusted models of stem volume and taper for *P. pseudostrobus* in *Nuevo León;* Tamarit-Urias *et al.* (2016), by modeling the stem profile of nine species of the *Pinus genus*, and obtaining projections according to the trends of the observed data. It also coincides with those reported by Tamarit *et al*. (2017) in the construction of a cubing system for individual trees for the *Quercus genus* in *Puebla* and those reported by Silva *et al.* (2018), who constructed compatible taper and commercial volume functions for three *Pinus* species in C*hihuahua*.

These tools generated at work can be complemented with the equations proposed by García *et al*. (2017), which propose to estimate the stem volume through the dimensions of the stump diameter in *Bursera simaruba* and *Metopium brownei* in the tropical forests of *Quintana Roo*, and can be taken as an update of the equations proposed by De los Santos (1976) and Patiño *et al.* (1994) for these two species.

* * *

Conclusions

The proposed volume models based on diameter at different heights above the stem are reliable to estimate the stem volume when using the first part of the equation, the volume ratio when using the second part, and the commercial volume when used together in these two species. This is a very significant advantage if it is compared with equations generated more than 30 years ago to estimate clean stem volume or total stem volume for large trees, which does not correspond to the diameters that are used today. With the mathematical structures proposed by species, it will be possible to build product distribution tables to project the timber yields of areas under forest use, according to the current conditions of the *Bursera simaruba* (*Chakáh*) and *Metopium brownei* (Black *Chechem*) species in tropical forests of the central and southern state of *Quintana Roo*, Mexico.

It is necessary to carry out this type of study in *Quintana Roo*, in order to cover at least the 30 main species that are most important from a commercial point of view.

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Conflict of interests

The authors declare no conflict of interests.

Contribution by author

Jonathan Hernández Ramos, Adrián Hernández Ramos and Xavier García Cuevas: study design, data collection and analysis, statistical adjustment and manuscript elaboration; Juan Carlos Tamarit Urias and Enrique Buendía Rodríguez: review, analysis and discussion of the document.

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