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Review article

Aspectos metodológicos para generar diagramas de manejo de la densidad de rodales con base en el índice de *Reineke*

Methodological aspects to generate density management diagrams based on the diagrams based on Reineke´s index

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Resumen

La densidad indica el grado de aglutinamiento de los árboles en un rodal, sus cambios afectan el rendimiento maderable y el tamaño de los individuos. El modelo de *Reineke* y su índice de densidad del rodal (IDR) se usan en la construcción de diagramas para manejar la densidad (DMD), el IDR define el límite superior del autoaclareo y el punto mínimo de ocupación total del sitio. A pesar de su importancia, en la construcción de un DMD es común que exista desconocimiento o poca claridad sobre aspectos metodológicos relevantes. El objetivo es presentar elementos teóricos básicos que el responsable del manejo forestal debe considerar para construir un DMD. Bajo el enfoque clásico de modelación de la función de *Reineke*, se abordan conceptos sobre densidad, el tipo y características de la fuente de información dasométrica por utilizar, la selección de sitios en alta densidad, el ajuste estadístico de la función, criterios metodológicos para desplazar y delimitar la línea de autoaclareo, y juicios para determinar las zonas de crecimiento de *Langsater*. Se resalta que la línea de autoaclareo puede delimitarse a partir de sitios de muestreo para inventario maderable, y los de densidad alta se seleccionan aplicando el enfoque del IDR máximo ($IDR_{máx}$); el método de mínimos cuadrados ordinarios lineales para ajustar la función de *Reineke* es robusto y corrige la heterocedasticidad. Para el crecimiento máximo en volumen, los rodales es factible manejarlos entre 35 y 65 % con respecto al $IDR_{máx}$. Los niveles óptimos de densidad deben determinarse por especie.

Palabras clave: Guía de densidad, índice de densidad del rodal, línea de autoaclareo, rodal coetáneo, sitios de muestreo, zonas de crecimiento.

Abstract

Density represents the degree of crowding of trees in a stand; their changes affect both the timber yield from a site and the sizes of the individual trees on it. Reineke´s model and its stand density index (SDI) are used in the construction of density management diagrams (DMDs). The SDI defines the upper limit for self-thinning and the minimum point of total site occupancy. Despite the importance of DMDs, in their construction there is often ignorance or lack of clarity about relevant methodological aspects. The objective of this paper is to present basic theoretical elements that those responsible for forest management should take into account to generate a DMD. Under the classical approach to modeling Reineke's function, concepts about density, the type and characteristics of the source of dasometric information to be used, this research addresses the selection of high-density sites, the statistical adjustment of the function, the methodological criteria to move and delimit the line of self-thinning, and the bases for determining the growth areas of Langsaeter. It emphasizes that the self-thinning line can be delimited by using sampling sites for timber inventory and that high-density sites are selected using the maximum SDI approach (SDI_{max}). The linear ordinary least squares method for adjusting Reineke's function is robust and corrects heteroscedasticity. For maximum volume growth, stands should be maintained between 35 and 65 % with respect to the SDI_{max}. Optimal density levels should be determined by species.

Key words: Stocking diagram, stand density index, self-thinning line, even-aged stand, sampling sites, growth areas.

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Introduction

Mexico, with 46 species of the *Pinus* genus (pine), is the world's largest center of diversity; its taxa make up the most important natural forests because they cover around 5 % of the national territory and 60 % of pine taxa are of commercial importance (Sánchez, 2008). In the period from 1990 to 2017, the total national timber production was, on average, 6.86 million m^3 of total logs, and pine contributed with 79.38 % (Semarnat, 2019). These attributes make the genus the most relevant for timber and the one with the highest economic potential; therefore, it is necessary to expand the knowledge that contributes to improve its technical management.

Within this context, it is relevant to have quantitative tools that may allow the application of intensive forestry and contribute to making informed decisions in order to achieve timber production objectives. In this regard, density management for controlling intraspecific competition is the main variable that the foresters can manipulate through thinnings, as intermediate forestry treatments (Daniel *et al.*, 1979; Smith *et al.*, 1997) that can be applied through density management diagrams (DMD).

In forests formed by coetaneous stands that are managed with some regular management system, such as the Forestry Development Method (FDM), it is important for forest managers to have technical elements and a clear knowledge that may allow them to build useful DMDs for diagnosing the level of competence, as well as for generating and prescribing thinnings.

It is particularly important to master in detail the fundamentals involved in the classical approach to the modeling and adjustment of Reineke's function, which is used to determine the self-thinning and to build DMDs. These are quantitative planning and assessment tools that help the foresters to rate and classify the level of site occupation and determine, if necessary, the required degree of intervention of the trees to be removed (Comeau *et al.*, 2010; Long and Shaw, 2012; Vospernik and Sterba, 2015). The DMDs are built with a mathematical stand level model, based on the concept of self-thinning that is plotted at a log scale, and which describes the density/size ratio. It includes four growth areas that indicate the stages of stand development for a particular species (Shaw and Long, 2007; Pretzsch, 2009; Salas and Weiskittel, 2018).

Despite the importance of the DMDs, the information on theoretical methodological aspects for building them are not sufficiently clear and are dispersed; in part, this is why the generation of this type of tool by taxon and ecoregion is reduced, and its availability to the foresters is limited. Therefore, the objective of this study was to present information on basic theoretical aspects and elements that the forest manager should consider to build a DMD, when using the classical Reineke´s density-size function modeling approach.

Fundamental concepts

The relevant concepts within the context of stand density management and the construction of a DMD with Reineke’s model (Reineke, 1933) are described below.

Stand density. In a forest site, stand density refers to the degree of space occupation that the trees have at a given time (Daniel *et al*., 1979; Clutter *et al*., 1983; Smith *et al*., 1997); it is a variable that through proper management helps to predict and improve some stand attributes. Density is manipulated to favorably influence the establishment and development of the species of interest; to improve the quality of wood, as well as the rate of diameter growth and the production of wood volume. Density can be expressed and assessed in absolute terms per unit area (number of living trees, basimetric area or volume), in which case the determination is direct and without reference to any other stand. It can also be expressed in relative terms by index values that determine the level of density and competence of a stand, such as Reineke’s stand density index (1933). In this form, density is expressed based on a previously selected standard density.

Reineke’s density-size function (1933). According to Smith *et al.* (1997), Pretzsch (2009) and Burkhart and Tomé (2012), it is a potential type relationship expressed as the number of trees per hectare of a stand (*N*), and the size of individuals, given by the quadratic mean diameter (*QMD*) (1).

$$N = \alpha QMD^{\beta} \qquad (1)$$

Where

α = Intercept parameter

β = Parameter of the slope

For ease of adjustment and reduction of heteroscedasticity, this function is used in its linear form by means of logarithms (ln) (2).

$$ln(N) = \alpha + \beta \, ln(QMD) \qquad (2)$$

Reineke (1933) established that the theoretical value of the slope for any species is -1.605.

Quadratic mean diameter (*QMD*). This is the diameter of the tree with the average basal area of the stand (*AB*, m^2 ha^{-1}), calculated with the expression (3) (Daniel *et al.*, 1979; Clutter *et al.*, 1983; Weiskittel *et al.*, 2011).

$$QMD = \sqrt{(40000/\pi)(AB/N)} \qquad (3)$$

Where:

π = The constant 3.1416

40 000 = It is used to express *AB* in m^2 when the normal diameter is expressed in cm

N = Number of trees ha^{-1}

Reference quadratic mean diameter (*QMDR*). It is the diameter that is predefined to estimate the SDI; it can be the value suggested by Reineke (1933), equivalent to 25 cm, or the value of the *QMD* observed in plots or sampling sites; even, any other value, since it is a self-reference model (Clutter *et al.*, 1983; Pretzsch, 2009; Burkhart and Tomé, 2012).

Reineke's Stand Density Index (*SDI*). This is the number of trees per hectare (or acre) that a stand can have for the predefined *QMDR* (Pretzsch, 2009; Weiskittel *et al.*, 2011; Burkhart and Tomé, 2012); it is calculated with the expression (4).

$$SDI = N(QMDR/QMD)^{-\beta} \tag{4}$$

An alternate and equivalent way of calculating it is through the expression (5)

$$SDI = N(QMD/QMDR)^{\beta} \tag{5}$$

When the value of β is unknown, it is possible to use the theoretical value 1605. The relative density is expressed in relation to the preset *QMDR*.

Maximum Stand Density Index (SDI_{max}). It is maximum density in terms of the number of trees per unit area that can exist in a population under self-thinning for a preset *QMDR* (example *QMDR*=25) and specific taxon (Pretzsch, 2009; Weiskittel *et al.*, 2011; Burkhart and Tomé, 2012). The SDI_{max} is estimated by the expression (6).

$$SDI_{max} = \alpha QMDR^{\beta}$$ (6) or its equivalent:

$$SDI_{max} = e^{(\alpha+\beta\ ln(QMDR))} \tag{7}$$

Where:

e = Exponential function

ln = Natural logarithm

Upper limit line for self-sunlighting (for maximum competition or simply self-sunlighting line) This is the line given by the maximum number of trees (N_{max}) for each QMD or diameter category, representing 100 % or the SDI_{max} (Pretzsch, 2009; Weiskittel *et al.*, 2011; Burkhart and Tomé, 2012), is determined by the expression (8).

$$N_{max} = \alpha QMD^{\beta}$$ (8) or its equivalent:

$$N_{max} = e^{(\alpha+\beta\ ln(QMD))}$$ (9)

Information sources for variables *N* and *QMD*

The mensuration information used in most of the classical works on density to delimit the maximum density line and to build a DMD should preferably come from permanent experimental plots, located in stands with extreme density and competition; a network of such plots allows monitoring, in time, the behavior of both crown closure and the occurrence of mortality periods (Weller, 1987; Zeide, 1987); however, due to long-term planning, the costs involved and the time required for the establishment and monitoring of the network, it is difficult to have sufficient mensuration data in the form of complete time series in the short and medium term (Pretzsch, 2006; Condés *et al.*, 2017). Some recent density studies utilize novel adjustment techniques such as boundary functions, using all available information from sampling sites or temporary plots (Lopes *et al.*, 2016; Quiñonez *et al.*, 2018; Salas and Weiskittel, 2018). However, it should always be ensured that the data set contains observations on maximum density.

Under the classical approach of working density with Reineke's model, another way of obtaining the information to be processed is by means of a directed sampling oriented to temporary plots, of circular, rectangular or with any other shape. In this case, selective stratified sampling is required; the useful strata are those corresponding to the stands with the highest population or density; which leads to the selection of stands with closed coverage and high densities (Valencia, 1994).

The plots will be located in pure and contemporaneous stands, with maximum density (normal or complete), without exhibiting physical damage, with healthy trees, and free of phytosanitary problems, preferably not intervened in the last 10 years; all the diameter categories, age classes and qualities of the season in which the species of interest is developed must be included. The plot area can vary according to the size of the trees; if they are adults, an area of 1 000 m^2 is recommended; if it holds a stage of growth and development (seedling, sapling, thicket, or pole), then the amount of trees is high and the area can be reduced, for example from 1 000 to 500 or even less m^2 (Valencia, 1994; Gezán *et al.*, 2007; Navarro *et al.*, 2011).

The most practical, economical, easy and fast alternative is to use the mensuration data information that come from classic sampling sites used in operational timber inventories, which are necessarily made for the purpose of developing and implementing management programs in a wide range of growth conditions (Weiskittel *et al.*, 2009). In this case, it is important to verify that the sample is representative of the study region and for the taxon of interest.

Navarro *et al.* (2011) state that appropriately selected high density sites are useful to determine the self-selected line, although a mixture of information from permanent and temporary sample units can be used; van Laar and Akça (2007), Vospernik and Sterba (2015) and Salas and Weiskittel (2018) ratify that, in the lack of remeasurement of permanent plots, mensuration information from temporary plots or inventory sites is adequate, after a selection treatment to delimit the self-thinning line.

These sites or plots replace the temporal succession of information from the permanent plots with a series of spatial-type mensuration information, distributed at different stages of development. Since they are only surveyed on one occasion, there is a collection of data on different stages of forest development with point measurements; therefore, a set of these sites successfully replaces a permanent plot with measurements staggered over time. However, they would have the disadvantage that it would not be possible to determine dynamic growth patterns, or mortality — essential information under some methodologies to delimit certain growth areas for the density guidelines.

High Density Site Selection

Within the context of traditional stand density analysis, not all sampling sites for operational inventory are necessarily established in extremely competitive, high-density stands. Only those that meet this condition are useful for conducting density studies, as well as for adjusting Reineke's function, defining the upper limit line of self-clearance, and constructing DMDs. Therefore, only the sites that meet this requirement and condition should be selected. For this purpose, we recommend using the method suggested by Solomon and Zhang (2002), known as the Maximum Stand Density Index Method (SDI_{max}), the steps of which are detailed below.

1. The density of each site expressed as the number of trees per site (*N*) is scaled to the hectare level.
2. For each site, the quadratic mean diameter (*QMD*) is calculated, using the expression (4), at the hectare level.
3. For each site, Reineke's *SDI* is calculated with the expression (5), *QMDR* = 25 cm and $\beta = 1.605$.
4. Of all the sites, the one with the highest *SDI* value (SDI_{max}) is located.
5. The relative density (*RD*) of each site is calculated by dividing its respective *SDI* value by the value corresponding to the site with the highest *SDI*:

$$RD = {}^{SDI}/_{SDI_{max}} \quad (9)$$

6. Those sites whose *RD* values are equal to or greater than 0.60 are selected; this ensures that they meet the condition of having high density and competition levels.

Adjusting Reineke's density-size function

The value of the slope is different from the originally proposed theoretical value of -1.605 (Pretzsch, 2006; VanderSchaaf and Burkhart, 2007; Comeau *et al.*, 2010); therefore, it must be estimated for specific regions and taxa. In this regard, different statistical methods have been developed to adjust Reineke's function and obtain the parameter estimators (Zhang *et al.*, 2005; Burkhart and Tomé, 2012; Zhang *et al.*, 2015; Salas and Weiskittel, 2018). The Linear Ordinary Least Square (OLS) and Stochastic Border Regression (SBR) techniques are the two main techniques utilized (Santiago *et al.*, 2013).

Under the classical modeling approach, the OLS method can be successfully used to adjust the function, is robust and recognized as the best for estimating the average condition of the random variable of interest (Zhang *et al.*, 2005; Comeau *et al.*, 2010), since it minimizes the sum of squares of the distances between the observed and predicted values; in addition, the potential problem of heteroscedasticity is minimized (Gezán *et al.*, 2007).

The average line generated with OLS passes through the middle of the observed data cloud; however, the interest is to determine the upper limit; therefore, it cannot be considered *per se* as a maximum density biological line derived from the density-size relationship (Zhang *et al.*, 2013); therefore, it is necessary to use some criteria that properly delimit the maximum density line at the upper boundary.

Methodological criteria for shifting the average line and charting the self-thinning line

The maximum density line (upper limit of the self-thinning) is defined at the upper border of the observed data cloud, as this is where the extreme density and competition are found, and therefore, the highest mortality of individuals. For this aim, there are different methodological criteria based on keeping the value of the slope parameter (β) fixed and increasing the value of the intercept parameter (α) of Reineke's function, which allows a maximum α (α_{max}) (Burkhart and Tomé, 2012; Santiago *et al.*, 2013). Thus, a new linear function is obtained that generates a straight line parallel to the average line, which, when displaced towards the upper border, will correspond to the maximum density line.

The main criteria with a statistical basis for increasing the value of the intercept parameter and generating the self-thinning line are described below. The application of the criteria will be exemplified using the information from the adjustment of Reineke's function (Table 1) in the expression (1) applied to 90 sampling sites of *Pinus montezumae* Lamb. of the Forest Management Unit (Umafor) 2103 *Teziutlán* Region, in the state of *Puebla*, Mexico.

Table 1. Parameters and statistics for adjusting Reineke's density-size function for *Pinus montezumae* Lamb. of Umafor 2103.

Parameter	Estimator	*SE*	Significance	*UL*	*MSE*	*RMSE* (σ)	$_{adj}R^2$
α	12.01457	0.216	<0.0001	12.44383	0.0659	0.25663	0.88
β	-1.74014	0.068	<0.0001	1.87535			

SE = Standard error; *UL* = The upper limit of the confidence interval; *MSE* = The mean square of the error; *RMSE* (σ) = The root of the *MSE* or residual standard error; $_{adj}R^2$ = The determination coefficient adjusted by the number of parameters.

A). Increase the intercept by 1.96 standard deviations to the model error (*RCME* or σ); in such a way that, asymptotically, there is only a 2.5 % probability of locating sites that exceed this maximum density line (Gezán *et al.*, 2007). Another option is proposed by Andenmatten (2019), who uses two standard deviations. This is established to ensure that the new line does not include outliers, which could occur if it is increased to three standard deviations.

B). Match the number of trees estimated by Reineke's function with the site with the highest *SDI*, when the expression (4) and a *QMDR* = 25 cm are applied (Valencia, 1994; Solomon and Zhang, 2002; Comeau *et al.*, 2010). Of the 90 sites, the one with the highest *SDI* had values of *N* = 2 670 and *QMD* = 13.70 cm. Then, from the expression (2) that corresponds to the linearized Reineke's function, the parameter α is cleared, and the *N* and *QMD* values of the site in question are substituted. Graphically, it should be observed that the displaced line intercepts the *N*-*QMD* point of the site that exhibited the highest *SDI*.

C). Use the average of the three residuals (errors) with the highest values (Zhang *et al.*, 2005; Burkhart and Tomé, 2012). The largest positive values corresponding to the three residuals are selected. In the example, they exhibited the values 0.41937, 0.40374 and 0.43036. The resulting average value is added to that of the intercept parameter. With this criterion, the maximum density line is placed in an average position between the three points of the maximum residuals. An alternative way is to use only the highest residual and run the line until it is equal to zero; this method is also known as corrected OLS.

D). Use the value of the intercept parameter corresponding to that of the upper limit (UL) of the confidence interval (CI) estimated at 95 % (Salas and Weiskittel, 2018).

Table 2 shows the method of calculation for each criterion, as well as the final values of α_{max} according to each criterion. It has been proven that the methodological criteria referred for correcting the value of the intercept are consistent for generating the absolute biological line of the density-size relationship (Solomon and Zhang, 2002; Comeau *et al.*, 2010).

Table 2. Values taken by the intercept of Reineke's function corrected according to each methodological criterion that displaces the adjusted average density line.

Criterion	Calculation	α_{max}
A	$\alpha_{max} = (1.96 * \sigma^2) + \alpha$	12.51756
B	$\alpha_{max} = Ln\,(2670) + \beta * Ln(13.7)$	12.44447
C	$\alpha_{max} = 0.41783 + 12.01457$	12.43240
D	$\alpha_{max} = 12.44383$	12.44383

Optimal stand growth area in volume

Based on evidence from various studies on tree mortality and competition (Drew and Flewelling, 1979; Long, 1985; Becerra, 1986; Dean and Baldwin, 1993; Jack and Long, 1996; Powell, 1999; Vargas, 1999; Gezán *et al.*, 2007; Shaw and Long, 2007; Navarro *et al*., 2011; Weiskittel *et al*., 2011; Long and Shaw, 2012; Müller *et al*., 2013; Hernández *et al*., 2013), it is assumed that the theoretical lines delimiting Langsaeter's growth areas can be determined in terms of relative density (percentages of the self-thinning line). This is because it is accepted that a certain interval of the SDI_{max} corresponds to a particular stage of stand development (Kumar *et al.*, 1995; Pretzsch, 2009) and each of these is equivalent to the growth areas of a DMD; which shows the importance of establishing correct relative density intervals to build a DMD. The lines that make up a DMD and their determination are presented below.

Line A. It is the self-thinning line and corresponds to 100 % or SDI_{max}. It determines the maximum density, and thus, the competition and maximum occupation of the site. The stands on this line are in a state of over-density; it is only desirable to maintain stands in this condition, prior to final stand harvest, to maximize total net timber production.

Line B. It corresponds to the lower limit of the area in which the self-thinning occurs and determines the beginning of mortality by competition since the site is fully occupied. This phenomenon starts between 50 and 60 % of the maximum density. Lines A and B form the growth area 4, called self-thinning or imminent mortality, which begins to be critical from a value of 75 %. In order to facilitate the generation of self-thinning scenarios and to have a wide range of density management, the lower limit of this area may be set at 65 %.

Line C. It determines the lower limit of occupation of the site and normally corresponds to the moment when the canopy closure begins to occur, partial competition begins, and, therefore, immediate mortality of the individuals does not occur. This line is obtained when the density varies from 25 to 35 %, with respect to the self-thinning line; the value of 35 % is commonly utilized for building a DMD. Between lines B and C lies the growth area No. 3, which corresponds to the maximum growth in volume per hectare of the stand, the occupation of the site is considered adequate, and the maximum and constant positive growth rate. Theoretically, the optimal range for this area is between 35 and 65 %.

The percentages are likely to change and be established based on the knowledge of the forester regarding the development and growth of the species of interest; so that full site occupancy and absence of competing mortality is achieved. Long (1985) suggests that to properly delimit the upper border of the growing area, the criterion of establishing a minimum acceptable level of individual tree vigor must be used; this should be related to the ability to respond quickly to thinning. A 40 % ratio of the live crown to the total height ratio of the tree may be adequate; furthermore, it corresponds to 50 % of the SDI_{max}. In order to apply this criterion, it must be taken into account that the live crown diminishes as the lower branches die, due to the increased relative density and the presence of natural pruning and crown closure.

Penner *et al.* (2006) suggest that growth area 3 corresponds to the condition of maximum periodic annual increment (PAI) of stand volume. The appropriate level of density can be determined when stability is observed in the PAI at normal diameter;

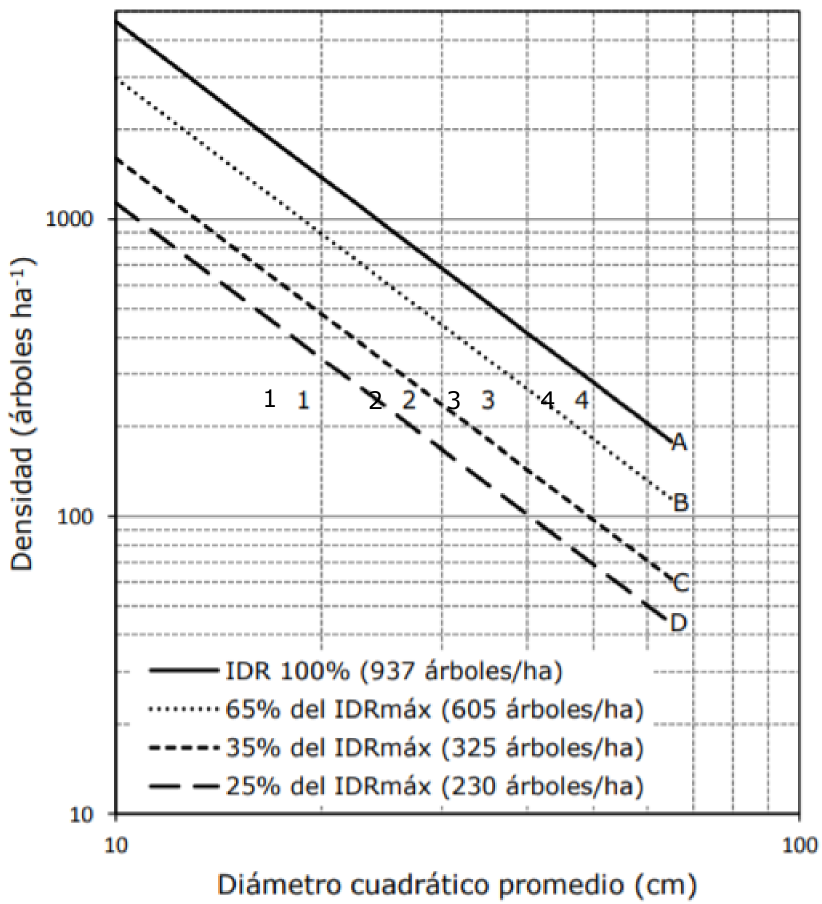
thus, for a specific diameter category, the PAI will reach a maximum and tend to stabilize; because, even if space increases, diametric growth will not increase. Otherwise, when the PAI reaches a minimum and tends to remain stable, it will indicate that the limit of survival and growth has been reached; therefore, if the density augments, the increase in diameter will not be reduced further. The fundamental principle for this area is that the silvicultural regimes applied should not allow the site to be wasted by excessive thinning and that growth should not be delayed by excessive density.

Line D. This is determined in order to know the minimum level of density that a stand must have so as to achieve minimum soil coverage; thus, the stand has the capacity to develop and reach full density at later stages. Generally, it is established at 25 % of the SDI_{max}. Lines C and D form the growth area 2 which corresponds to the transition area and varies from 25 to 35 %.

Growth area 1 is the underutilization of the site. The trees grow freely, without competition, and in isolation and independently of each other; since the space is large for their size, it is delimited between zero and 25 % in relation to the SDI_{max}; the growth rate in volume in this area is constant and proportional to density.

The four growth areas referred to above form bands of relative densities and give rise to a DMD. In order to promote optimal growth of each stand for timber production purposes, they should be maintained in area 3 by means of thinning density management (Drew and Flewelling, 1979; Jack and Long, 1996; Long and Shaw, 2012).

Figure 1 shows the DMD generated with the adjustment of Reineke's function (Table 1) and the intercept corrected with criterion B. The referred growth lines that delimit the growth areas are shown. The maximum density line indicates the maximum number of trees for different *QMDs* or diameter category centers at an interval that depends on the nature of maximum growth in diameter of the species of interest.



Densidad (árboles ha^{-1}) = Density (trees ha^{-1}); Diámetro cuadrático promedio = Quadratic mean diameter; *IDR* = *SDI*; $IDR_{máx}$ = SDI_{max}.

Figure 1. DMD with the lines that delimit the growth areas.

Final considerations

The support provided by DMDs makes it easier to make decisions to define thinning regimes, making them an important quantitative forest management tool. What the DMD suggests should be considered only as a reference and not as a strict rule. The decision to thin, as well as the type, intensity and periods between consecutive thinnings is the responsibility of the foresters, whose decision is based on the particular situation of each stand and on the productive objective of the species.

Proper density management as a function of the application of intermediate cuttings has a direct effect on the attributes of the individuals that make up the stand, such as average diameter, taper, average crown length, branch size, vigor, health, and, primarily, volume. These attributes directly influence the quality and quantity of wood and therefore its commercial value. From the above, it is evident that it is important that the technician in charge of forest management knows how to build and dispose of DMDs that are based on Reineke's function, when the classical modeling approach is used.

DMDs built with Reineke's model are more accurate, since they use the quadratic diameter, calculated based on the normal diameter, and this is measured directly on the trees, unlike other indices that are considered less stable, such as Yoda's, which uses the average volume estimated indirectly through volume models, or the relative spacing index that uses the total height, which is obtained indirectly, or estimated with some allometric model. In both cases, a larger margin of error is incurred, and therefore its accuracy and reliability may be lower. The initially generated DMDs are basic, since only two stand attributes are represented in a two-dimensional plane; however, they are practical since they facilitate reading and interpretation to diagnose the density and competence status of the stands. The subsequent incorporation of isolines of other stand variables, such as dominant height or site index, basal area, volume, or other, into the DMD will allow for the development of more complex, yet more comprehensive and complete, density diagrams.

Conflict of interests

The authors declare no conflict of interest.

Contribution by author

Juan Carlos Tamarit-Urias: documentary research of specialized literature, statistical analysis of the mensuration information, drafting and editing of the manuscript; Gerónimo Quiñonez-Barraza and Jonathan Hernández-Ramos: review and editing of the document.

References

Andenmatten, E. 2019. Autorraleo: alternativa para determinar la pendiente de la relación lineal tamaño-densidad, mediante su impacto en la estimación del volumen del rodal. Bosque 40(2): 153-162. Doi: 10.4067/S0717-92002019000200153.

Becerra L., F. 1986. Determinación de una guía de densidad para *Pinus patula* Schdl. en Chignahuapan - Zacatlán, Pue. Tesis maestría. Colegio de Postgraduados. Chapingo, Edo. de Méx., México. 82 p.

Burkhart, H. E. and M. Tomé. 2012. Modeling forest trees and stands. Springer. New York, NY, USA. 457 p.

Clutter, J. L., J. C. Fortson, L. V. Pienaar, G. H. Brister and R. L. Bailey. 1983. Timber management: a quantitative approach. John Wiley & Sons Inc. New York, NY, USA. 333 p.

Condés, S., P. Vallet, K. Bielak, A. Bravo-Oviedo, L. Coll, M. J. Ducey, M. Pach, H. Pretzsch, H. Sterba, J. Vayreda and M. del Río. 2017. Climate influences on the maximum size-density relationship in Scots pine (*Pinus sylvestris* L.) and European beech (*Fagus sylvatica* L.) stands. Forest Ecology and Management 385: 295-307. Doi: 10.1016/j.foreco.2016.10.059.

Comeau, P. G., M. White, G. Kerr and E. S. Hale. 2010. Maximum density-size relationships for Sitka spruce and coastal Douglas-fir in Britain and Canada. Forestry 83(1): 461-468. Doi: 10.1093/forestry/cpq028.

Daniel, T. W., A. J. Helms and S. F. Baker. 1979. Principles of silviculture. 2nd Edition. McGraw-Hill. New York, NY, USA. 500 p.

Dean, T. J. and V. C. J. Baldwin. 1993. Using a density-management diagram to develop thinning schedules for loblolly pine plantations. Research Paper SO-275. USDA, Forest Service, Southern Forest Experiment Station. New Orleans, LA, USA. 7 p.

Drew, T. J. and W. J. Flewelling. 1979. Stand density management: an alternative approach and its application to Douglas-fir plantations. Forest Science 25: 518-532. Doi: 10.1093/forestscience/25.3.518.

Gezán S., A., A. Ortega y E. Andenmatten. 2007. Diagramas de manejo de densidad para renovales de roble, raulí y coigüe en Chile. Bosque 28(2): 97-105. Doi: 10.4067/S0717-92002007000200002.

Hernández R., H., J. J. García M., H. J. Muñoz F., X. García C., T. Sáenz R., C. Flores L. y A. Hernández R. 2013. Guía de densidad para manejo de bosques naturales de *Pinus teocote* Schlecht. *et* Cham. en Hidalgo. Revista Mexicana de Ciencias Forestales 4(19): 62-76. Doi: 10.29298/rmcf.v4i19.379.

Jack, B. S. and M. N. Long. 1996. Linkages between silviculture and ecology: an analysis of density management diagrams. Forest Ecology and Management 86: 205-220. Doi: 10.1016/S0378-1127(96)03770-X.

Kumar, M. B., N. Long, J. and P. Kumar. 1995. A density management diagram for teak plantations of Kerala in peninsular India. Forest Ecology and Management 74: 125-131. Doi: 10.1016/0378-1127(94)03499-M.

Long, J. N. 1985. A practical approach to density management. Forestry Chronicle 61: 23-27. Doi: 10.5558/tfc61023-1.

Long, N. J. and J. D. Shaw. 2012. A density management diagram for even-aged Sierra Nevada mixed-conifer stands. Western Journal of Applied Forestry 27(4): 187-195. Doi: 10.5849/wjaf.11-036.

Lopes P. E., N. Calegario, G. Saraiva N., E. de Almeida M. and J. de Almeida A. 2016. Estimate of stand density index for *Eucalyptus urophylla* using different fit methods. Revista Árvore 40(5): 921-929. Doi: 10.1590/0100-67622016000500016.

Müller, U. B., R. Rodríguez y P. Gajardo. 2013. Desarrollo de una guía de manejo de la densidad en bosques de segundo crecimiento de roble (*Nothofagus obliqua*) en la región del Biobío. Bosque 34(2): 201-209. Doi: 10.4067/S0717-92002013000200009.

Navarro C., M. Herrera, F. Drake y P. Donoso. 2011. Diagrama de manejo de densidad y su aplicación a raleo en bosques de segundo crecimiento de *Drimys winteri* en el sur de Chile. Bosque 32(2): 175-186. Doi: 10.4067/S0717-92002011000200008.

Penner, M., E. D. Swift, R. Gagnon and J. Brissette. 2006. A stand density management diagram for balsam fir in New Brunswick. The Forestry Chronicle 82(5): 700-711. Doi: 10.5558/tfc82700-5.

Powell, C. D. 1999. Suggested stocking levels for forest stands in northeastern Oregon and southeastern Washington: an implementation guide for the Umatilla National Forest. USDA, Forest Service. F14-SO-TP-03-99. Pendleton, OR, USA. 72 p.

Pretzsch, H. 2006. Species-specific allometric scaling under self-thinning: evidence from long-term plots in forest stands. Oecologia 146: 572-583. Doi: 10.1007/s00442-005-0126-0.

Pretzsch, H. 2009. Forest dynamics, growth and yield: from measurement to model. Springer-Verlag Berlin. Heidelberg, Germany. 664 p.

Quiñonez B., G., J. C. Tamarit U., M. Martínez S., X. García C., H. M. de los Santos P. and W. Santiago G. 2018. Maximum density and density management diagram for mixed-species forests in Durango, Mexico. Revista Chapingo Serie Ciencias Forestales y del Ambiente 24(1): 73-90. Doi: 10.5154/r.rchscfa.2017.09.056.

Reineke, L. H. 1933. Perfecting a stand-density index for even-aged forests. Journal of Agricultural Research 46: 627-638.
https://naldc.nal.usda.gov/download/IND43968212/PDF (7 de enero de 2020).

Salas, E. C. and A. R. Weiskittel. 2018. Evaluation of modeling strategies for assessing self-thinning behavior and carrying capacity. Ecology and Evolution 8: 10768-10779. Doi: 10.1002/ece3.4525.

Sánchez G., A. 2008. Una visión actual de la diversidad y distribución de los pinos de México. Madera y Bosques 14(1): 107-120. http://www.scielo.org.mx/pdf/mb/v14n1/v14n1a8.pdf (8 de enero de 2020).

Santiago G., W., H. M. De los Santos P., G. Ángeles P., J. R. Valdez L., D. H. Del Valle P. y J. J. Corral R. 2013. Auto-aclareo y guías de densidad para *Pinus patula* mediante el enfoque de regresión de frontera estocástica. Agrociencia 47: 75-89. http://www.scielo.org.mx/pdf/agro/v47n1/v47n1a7.pdf (8 de enero de 2020).

Secretaría del Medio Ambiente y Recursos Naturales (Semarnat). 2019. Producción forestal maderable y no maderable. https://apps1.semarnat.gob.mx:445/dgeia/indicadores17/conjuntob/indicador/07_forestales/7_2.html (2 de diciembre de 2019).

Shaw, D. J. and J. N. Long. 2007. A density management diagram for longleaf pine stands with application to red-cockaded woodpecker habitat. Southern Journal of Applied Forestry 31(1): 28-38. https://www.fs.fed.us/rm/pubs_other/rmrs_2007_shaw_j001.pdf (6 de enero de 2020).

Smith, D. M., B. C. Larson, M. J. Kelty and P. M. S. Ashton. 1997. The practice of silviculture: Applied forest ecology. Ninth Edition. John Wiley & Sons Inc. New York, NY, USA. 537 p.

Solomon, S. D. and L. Zhang. 2002. Maximum size-density relationships for mixed softwoods in the northeastern USA. Forest Ecology and Management 155: 163-170. Doi: 10.1016/S0378-1127(01)00556-4.

Valencia V., J. 1994. Utilización del índice de densidad de Reineke en *Pinus douglasiana* en Atenquique, Jalisco. Revista Ciencia Forestal en México 19(76): 51-75.

VanderSchaaf, C. L. and E. H. Burkhart. 2007. Comparison of methods to estimate Reineke's maximum size-density relationship. Forest Science 53(3):435-442. Doi: 10.1093/forestscience/53.3.435.

van Laar, A. and A. Akça. 2007. Forest Mensuration. Springer. Dordrecht, The Netherlands. 383 p.

Vargas L., B. 1999. Caracterización de la productividad y estructura de *Pinus hartwegii* Lindl. en tres gradientes altitudinales en el cerro Potosí, Galeana, Nuevo León. Tesis maestría. Universidad Autónoma de Nuevo León, Facultad de Ciencias Forestales. Linares, NL., México. 93 p. http://eprints.uanl.mx/7905/1/1020125428.PDF (24 de junio de 2020).

Vospernik, A. and H. Sterba. 2015. Do competition-density rule and self-thinning rule agree? Annals of Forest Science 72(3): 379-390. Doi: 10.1007/s13595-014-0433-x.

Weller, D. E. 1987. A reevaluation of the -3/2 power rule of plant self-thinning. Ecological Monographs 57: 23-43. Doi: 10.1007/s13595-014-0433-x.

Weiskittel, A., P. P. Gould and H. Temesgen. 2009. Sources of variation in the self-thinning boundary line for three species with varying levels of shade tolerance. Forest Science 55(1): 84-93. Doi: 10.1093/forestscience/55.1.84.

Weiskittel, A. R., D. W. Hann, J. A. Kershaw and J. K. Vanclay. 2011. Forest growth and yield modeling. First Edition. John Wiley & Sons, Ltd. Chichester, West Sussex, UK. 415 p.

Zeide, B. 1987. Analysis of the 3/2 power law of self-thinning. Forest Science 33: 517-537. Doi: 10.1093/forestscience/33.2.517.

Zhang, L., H. Bi, J. Gove and L. Heath. 2005. A comparison of alternative methods for estimating the self-thinning boundary line. Canadian Journal of Forest Research 35: 1507-1514. Doi: 10.1139/x05-070.

Zhang, J., W. W. Oliver and F. R. Powers. 2013. Reevaluating the self-thinning boundary line for ponderosa pine. Canadian Journal of Forest Research 43: 963-971. Doi: 10.1139/cjfr-2013-0133.

Zhang, X., J. A. Zhang and A. Duan. 2015. A hierarchical bayesian model to predict self-thinning line for chinese fir in southern China. Plos One 10(10): e0139788. Doi: 10.1371/journal.pone.0139788.