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Article

Relaciones alométricas para plantaciones de *Pinus patula* Schiede ex Schltdl. *et* Cham. en el Estado de México

Allometric relations for *Pinus patula* Schiede Schltdl. *et* Cham. plantations in Mexico State

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Resumen

Las relaciones alométricas son los ajustes fenotípicos que manifiestan las especies a las condiciones ambientales donde se desarrollan. Los objetivos del presente estudio fueron: 1) ajustar modelos alométricos entre las variables de diámetro de tocón (*dt*), diámetro normal (*d*), altura total (*h*) y el volumen (*V*) para plantaciones forestales comerciales (PFC) de *Pinus patula* en las Regiones VI y VII del Estado de México; y 2) evaluar la diferencia entre los volúmenes resultantes de las ecuaciones empleadas por técnicos forestales del Estado de México. Se emplearon 1 825 datos de *dt*, *d* y *h* provenientes de 65 sitios de muestreo, para el ajuste de 13 modelos alométricos mediante máxima verosimilitud. La evaluación estadística se realizó con la significancia de los parámetros (p<0.05), la raíz del cuadrado medio del error, el coeficiente de determinación ajustado y sesgo. La regresión mostró parámetros significativos y errores estándar inferiores a 1.5; cuando se empleó el *dt* como variable independiente, se explica 54.7 % y 87 % del *h* y *V*, respectivamente; mientras que, si se usa el *d* para la relación con *h* y *V* se obtuvo 57.0 % y 91.9 %, respectivamente. Los modelos resultaron confiables para incluirlos en sistemas de crecimiento y rendimiento, o en la elaboración de planes de manejo de las PFC; además, la expresión propuesta por la Protectora de Bosques del Estado de México (Probosque) representa la mejor alternativa para la proyección del rendimiento maderable para *Pinus patula.*

Palabras clave: Aprovechamiento maderable, crecimiento, modelos matemáticos, plantaciones comerciales, rendimiento, variables dasométricas.

Abstract

Allometric relationships are the phenotypic adjustments of species to the environmental conditions where they develop. The objectives were 1) to fit allometric models between the variables stump diameter (*dt*), normal diameter (*d*), total height (*h*) and volume (*V*) for commercial forest plantations (CFP) of *Pinus patula* in Regions VI and VII from Mexico State, and 2) to evaluate the differences between the volumes resulting from the volume equations used by the forestry technicians of the State of Mexico. 1 825 *dt*, *d* and *h* data from 65 sampling sites were used to fit 13 allometric models using maximum likelihood. The statistical evaluation was based on the significance of the parameters (p<0.05), the root mean square error, the adjusted coefficient of determination, and the bias. The regression showed significant parameters and standard errors below 1.5 when using the *dt* as an independent variable, 54.7 % and 87 % of the *h* and *V* are explained, and 57.0 % and 91.9 % of the *h* and the *V*, respectively, when using the *d*. The models were reliable enough to be included in a growth and yield system or in the elaboration of CFP management plans. Furthermore, the expression proposed by PROBOSQUE (1990) constitutes the best alternative for the projection of timber yield for *P. patula.*

Keywords: Timber exploitation, growth, Mathematical models, commercial plantations, yield, mensuration variables.

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Introduction

Nearly 269 600 ha of commercial forest plantations have been established in Mexico (CFP) (Semarnat-Conafor, 2014a), reducing the pressure on the natural forests, promoting private investment and social development (Semarnat-Conafor, 2014b), and favoring the reconversion of degraded or unproductive areas into woodland (Probosque, 2019).

In the State of Mexico, the establishment of forest plantations (FP) has exhibited an upward trend; 65 ha were reported by 2003; 504 ha, by 2008, and approximately 537 ha by 2014, with a total of 3 070 ha of FP established between 2000 and 2014 (Semarnat-Conafor, 2015a; Probosque, 2019). Nearly 81 % of the forest plantations have been established with species of the *Pinus* genus (Semarnat-Conafor, 2015b), which must be managed using quantitative forestry tools.

Knowledge of the allometric relationships between the species and the environment in which they grow is essential for an adequate management ofFPs because they influence their growth and development (Semarnat- Conafor, 2014b).

Allometric relationships are a reflection of the adaptations of the species to the environmental conditions of the place where they develop (Niklas, 1995; Gildardo *et al.*, 2011) and can be quantified through a regression analysis and represented mathematically by means of allometric equations (Diéguez *et al.,* 2003; Corral *et al.*, 2007; Picard *et al.*, 2012).

In the forest area, allometric relationships have been modeled between the stump diameter (*sd*) and the normal diameter (*d*) (Benítez *et al.,* 2004;Pompa *et al.*, 2009), the *sd* and the total height (*h*) (Diéguez *et al*., 2009; Hernández *et al.*, 2017b; Hernández *et al.* 2018), the *d* and the volume (*V*) (Martínez and Acosta, 2014; García *et al.*, 2017), biomass (*B*) (Návar *et al.*, 2013) or carbon (*C*) (Martin *et al.*, 1998; Méndez *et al.*, 2011).

In the State of Mexico, the rate of establishment of CFPs has increased and has played an important role in the development and growth of the forest sector (Probosque, 2019). However, there are certain management problems, such as clandestine felling, loss of plantations due to meteorological causes or health issues, which make it necessary to have quantitative tools for assessing the volumetric losses in these crops. Thus, the objectives of the present study were 1) to adjust allometric models between the variables of commercial interest (*sd*, *d*, *h* and *V*) for *Pinus patula* Schiede ex Schltdl. *et* Cham. at CFPs in Regions VI and VII of the State of Mexico, and 2) to assess the differences between the volumes resulting from the equations used by the forest technicians of the State of Mexico for the *Pinus* genus.

Materials and Methods

The study area is located on the Neovolcanic Axis, in the forest regions delimited by Probosque: *Coatepec Harinas* (R-VI) and *Valle de Bravo* (R-VII), in the State of Mexico, which include *Coatepec de Harinas* and *Ocuilán* municipalities, for R-VI, and *Amanalco, Villa de Allende* and *Villa Victoria*, for R-VII (Cervantes *et al.,* 1990; Probosque, 2019). The altitude ranges between 2 000 and 2 850 masl; the climate is temperate (Cw); the precipitation is 1 200 mm, and the mean temperature, 14.5 °C (Inegi, 2016). The soil type is Andosol (CCT, 2010).

The sample was established in 90.8 ha distributed in 11 *P. patula* CFPs aged 4 to 20 years, with a 3 m × 3 m spacing and a mean density of 950 plants ha^{-1}. Eight CFPs are located in R-VII, and three, in R-VI. The polygons of each CFP were constructed with Google Earth® Pro, version 7.3.2.5491 and QGIS Madeira®, version 3.4.13. The sites were 250 m^2 in a rectangular shape, using a systematic sampling and a 100 m grid, at whose intersections they were established with the QGIS tool known as "regular points". In order to avoid the border effect, only sites located at a distance from the boundaries of the plantation were considered.

65 sites were assessed, in which the altitude and the exposure, as well as the stump diameter (*sd*), and normal diameter (*d*) were recorded with a Haglöf Mantax Blue® aluminum caliper; and total height (*h*) of each individual were recorded with a Pm-5/1520 Pc *Opti Height Meter®* clinometer. The individual volume was estimated using three methods:

a) Through the expressions of the shape factor:

$$V1 = (0.7854 \cdot d^2) \cdot h \cdot sf \qquad (1)$$

b) With the method proposed in the Second Dasonomic Study of the State of Mexico (Sedemex) (Probosque, 1990):

$$V2 = e^{((-9.7753)+(2.04668*Ln(d))+(0.81083*Nl(h)))} \qquad (2)$$

c) Using the method generated by Vargas *et al.* (2017):

$$V3 = 0.0000424 \cdot d^{1.8770286} \cdot h^{1.0933022} \qquad (3)$$

Where:

V = Stem volume (m^3)

d = Normal diameter at 1.30 m (m)

π = Constant of 3.1416

sf = shape factor (0.77)

h = Total (m)

e = Exponential function (2.718281828)

Nl = Natural logarithm

The observed data were captured in a scatter plot of the dispersion between the variables *sd-d*, *sd-h*, *sd-V1*, *sd-V2*, *sd-V3*, *d-h*, *d-V1*, *d-V2*, and *d-V3*; this allowed identifying atypical or aberrant values, which were then deleted from the database. The mathematical models were adjusted with the resulting screened information and after verifying the kurtosis.

13 allometric models were assessed for the *sd-d, sd-h, d-h, sd-V*, and *d-V* relationships were reported in the specialized literature (Huang *et al.*, 1992; Pompa *et al.*, 2009; Hernández *et al.*, 2015; Hernández *et al.*, 2017b; García *et al.,* 2017) (Table 1). The models were adjusted using the full information maximum likelihood technique (FIML) in the SAS® 9.2 software (SAS Institute In., 2008).

Table 1. Adjusted allometric models for *Pinus patula* Schiede ex Schltdl. *Et*

Allometric relationship	Model	Identifier
sd-d	$d = \beta_0 + \beta_1 sd$	4
	$d = \beta_0 + sd^{\beta_1}$	5
	$d = \beta_0 sd^{\beta_1}$	6
sd-h	$h = \beta_0\, e^{\beta_1\left(\frac{1}{dt}\right)}$	7
	$h = \beta_0 + \beta_1(\beta_1 sd)$	8
	$h = e^{\beta_0 + \beta_1 lnln\,(sd)}$	9
sd-V	$V = \beta_0 + \beta_1 e^{\beta_2\left(\frac{1}{dt}\right)}$	10
	$V = \beta_0 + \beta_1(sd) + (\beta_2(sd)^2) + 1.3$	11
d-h	$h = \beta_0 + \beta_1 d + \beta_2 d^2$	12
	$h = e^{(\beta_0 + \beta_1 d)}$	13
	$h = \beta_0 + \beta_1 nl(d)$	14
d-V	$V = \dfrac{d^2}{(\beta_0 + \beta_1 d)^2}$	15
	$V = e^{\left(\beta_0 + \beta_1 \frac{1}{d}\right)}$	16

Cham. forest plantations in the State of Mex*Sd* (or 1/*dt)* = Stump diameter (cm); *d* = Normal diameter (cm); *h* = Total height (m); *V* = Volume (m^3); B_0, B_1, B_2 = Parameters to be estimated; *Nl*= Natural logarithm; *e*=Exponential function.

The best model was selected based on the significance of the estimators ($p > 0.05$), the least square mean error (LSME) and the maximum adjusted determination coefficient (R^2_{Adj}). Furthermore, the normality and the distribution of the residuals were verified using the Shapiro-Wilk (SW) test and the graphic tendency of the observations, respectively (Huang, 1992; García *et al*., 2017). The accuracy of the estimations using the best models for each variable was assessed with the bias (*E*) (García *et al*., 2017; Corral *et al.*, 2019), and the estimations *versus* the observed data chart (García *et al.*, 2017).

The three estimated volumes (*V1*, *V2* and *V3*) were compared by the model selected as the best, based on the differences in the obtained biases and statistically with a *t* test for independent samples at a confidence level of 95 % (p=0.05), contrasting *V1*-*V2*, *V1*-*V3*, and *V2*-*V3* (Di Rienzo *etal.,* 2008).

Results

The descriptive statistics of the sample indicate that the *sd* exhibits a range of values between 6 and 42 cm, while the *d* ranges from 3 to 40 cm, with a maximum height of 25 m; the difference in the volume was 0.979 m^3 (*V1*-*V2*) when the equations were used. The kurtosis of the registered data (independent variables) indicates a normal distribution, without deviation problems (Table 2).

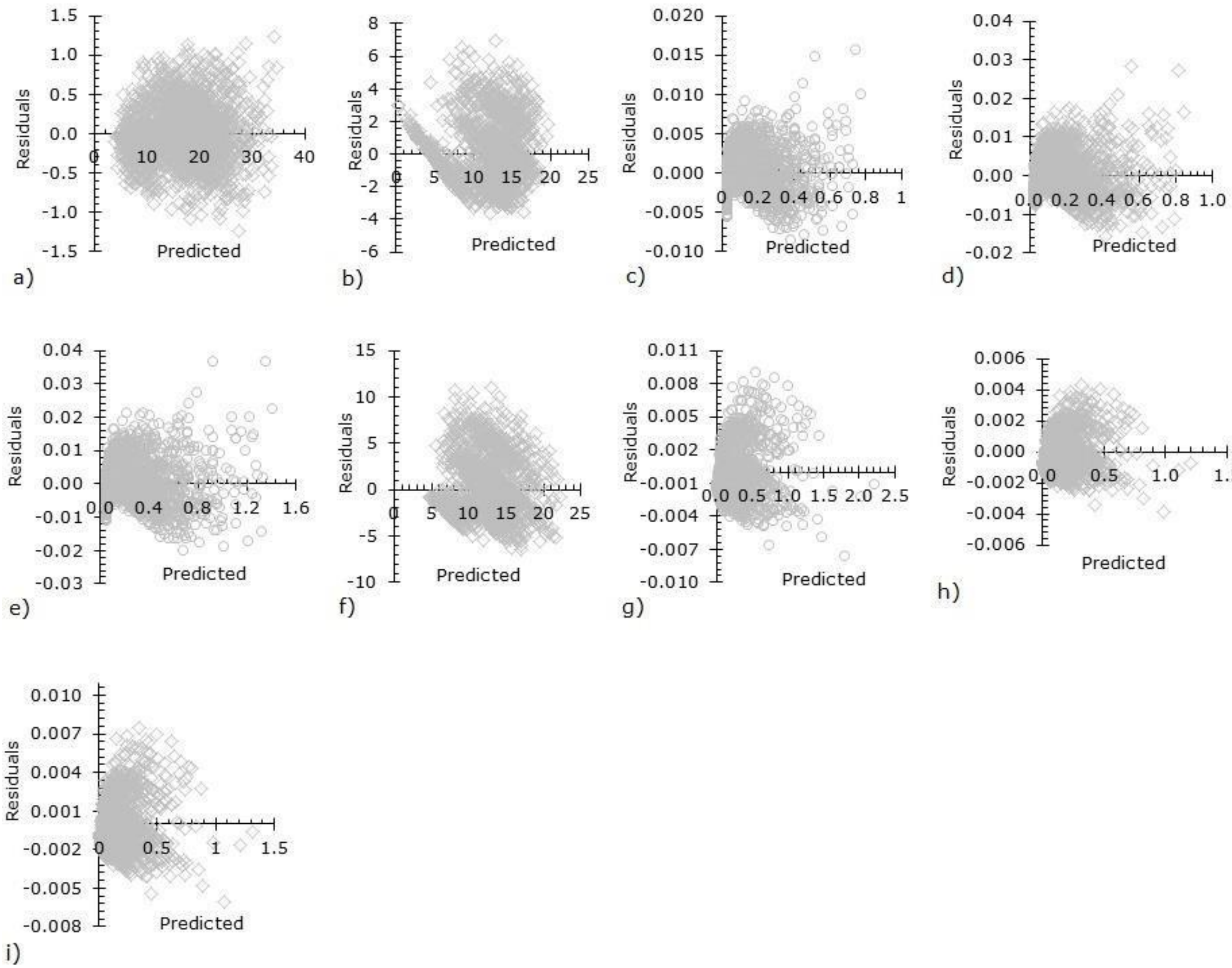
Table 2. Basic statistics of data observed in *Pinus patula* Schiede ex Schltdl. *Et* Cham.plantations in the State of Mexico, Mexico.

Statistic/ Variable	Stump diameter (cm)	Normal diameter (cm)	Total height (m)	Volume 1 (m^3)	Volume 2 (m^3)	Volume 3 (m^3)
Number of observations	1 825	1 825	1 825	1 825	1 825	1 825
Mean	20.99	17.40	12.19	0.284	0.165	0.176
Maximum	41.20	39.20	25.00	2.137	1.158	1.278
Minimum	6.00	3.40	4.00	0.002	0.002	0.001
Variance	46.98	37.53	21.46	0.066	0.020	0.024
Standard deviation	6.85	6.12	4.63	0.258	0.142	0.157
Kurtosis index	-0.39	-0.18	-0.56	5.394	4.658	5.032

In a first fit, when verifying the assumptions of the regression in the variables of interest, normality was observed (SW>0.93) for all models; however, the tendency of the residuals in the charts exhibited heteroscedasticity issues. Variance weighting functions were utilized in the solution of this problem (Prodan *et al.*, 1997).

The *sd-d, sd-h* and *d-h* relationships were weighted using the formula: $Residual/(x^{\varphi})^{0.5}$, and the *sd-V1, sd-V2, sd-V3, d-V1, d-V2* and *d-V3* relationships, using the $Residual/((1/x)^{\varphi})^{0.5}$ formula (Crecente *et al.*, 2009; García *et al*., 2017; Hernández *et al.,* 2017a). This function, based on an exponential equation, was applied according to the methodology suggested

by Harvey (1976), where the utilized variable is interpreted with x and φ, derived from the linear regression of the natural logarithm (*Nl*) of the residuals in the dependent variable based on *Nl(x)*; with this procedure, the residuals were homoscedastic in all the adjustments (Figure 1) and SW > 0.93.



a) Normal diameter; b) Height; c) Volume 1; d) Volume 2; and e) Volume 3 based on the stump diameter; and for f) Height; g) Volume 1; h) Volume 2; and i) Volume 3 (k) based on the normal diameter.

Figure 1. Distribution of the residuals resulting from the statistical adjustment in the allometric relationships for commercial forest plantations of *Pinus patula* Schiede ex Schltdl. *et* Cham. in the State of Mexico.

Once the heteroscedasticity problems were corrected, the results of the fit of the models utilized for the allometric relationship between *d*, *h*, *V1, V2* and *V3* based on *sd* account for 96.0, 54.7, 86.6, 89.1 and 85.2 % of the variability (R^2_{Adj}), respectively, and for 57.0, 91.5, 94.5 and 89.8 %, respectively, in the relationship between *d* and *h*, *V1*, *V2* and *V3* (Table 3). Significant parameters with a 95 % confidence interval for the best models and standard errors below 1.5 in all cases.

Table 3. Statistics for the adjustment and value of the parameters in allometric relationships of the stump diameter (*ds*), the normal diameter (*d*), the total height (*h*), and the estimated volume (*Vi*) for forest plantations of *Pinus patula* Schiede ex Schltdl. *et* Cham. in the State of Mexico.

Variable	Model	RMSE	$_{Adj}R^2$	Parameter	Estimator	*Ase*	T value	Pr>t
sd-d	4	1.498	0.960	β_0	-1.344670	0.1040	-12.97	<0.0001
				β_1	0.875760	0.0040	210.80	<0.0001
	5	1.463	0.960	β_0	-1.905270	0.0842	-22.62	<0.0001
				β_1	0.966882	0.0011	850.2	<0.0001
	6	1.528	0.959	β_0	0.639455	0.0120	53.20	<0.0001
				β_1	1.077157	0.0056	189.38	<0.0001
sd-h	7	9.711	0.547	β_0	27.33707	0.5581	48.98	<0.0001
				β_1	-15.8681	0.4427	-35.85	<0.0001
	8	9.811	0.543	β_0	1.73889	0.2560	6.79	<0.0001
				β_1	0.70583	0.0080	86.29	<0.0001
	9	9.716	0.547	β_0	-0.01125	0.0663	-0.17	0.8653
				β_1	0.828473	0.0210	39.54	<0.0001
sd-V	10.*V1*	0.008	0.866	β_0	0.051621	0.0080	6.41	<0.0001
				β_1	13.1044	0.4632	28.29	<0.0001
				β_2	-93.3043	1.4997	-62.22	<0.0001
	10.*V2*	0.002	0.891	β_0	0.03006	0.0040	7.53	<0.0001
				β_1	6.4056	0.1900	33.79	<0.0001
				β_2	-88.6921	1.2650	-70.1	<0.0001

	10.*V3*	0.003	0.852	β_0	0.03208	0.0052	6.07	<0.0001
				β_1	7.412401	0.2817	26.32	<0.0001
				β_2	-90.9122	1.6063	-56.6	<0.0001
	11.*V1*	0.009	0.858	β_0	2.211235	0.0201	109.9	<0.0001
				β_1	-79.5617	0.6700	-118.8	<0.0001
	11.*V2*	0.002	0.883	β_0	1.503327	0.0173	86.77	<0.0001
				β_1	-75.2011	0.5761	-130.6	<0.0001
	11.*V3*	0.003	0.844	β_0	1.643419	0.0214	76.62	<0.0001
				β_1	-77.229	0.7055	-109.5	<0.0001
d-h	12	9.229	0.570	β_0	-0.08326	0.6237	-0.13	0.8938
				β_1	0.919122	0.0684	13.43	<0.0001
				β_2	-0.01031	0.0017	-5.75	<0.0001
	13	10.49	0.511	β_0	1.776094	0.0181	98.26	<0.0001
				β_1	0.040935	0.0008	50.57	<0.0001
	14	9.335	0.565	β_0	-10.4242	0.5820	-17.9	<0.0001
				β_1	8.20582	0.2050	40.04	<0.0001
d-V	15.*V1*	0.005	0.915	β_0	42.54767	0.2371	179.45	<0.0001
				β_1	-0.41312	0.0077	-53.67	<0.0001
	15.*V2*	0.001	0.945	β_0	54.2885	0.2430	223.1	<0.0001
				β_1	-0.47138	0.0080	-59.18	<0.0001
	15.*V3*	0.002	0.898	β_0	53.15026	0.3405	156.09	<0.0001
				β_1	-0.48248	0.0112	-43.04	<0.0001
	16.*V1*	0.006	0.904	β_0	2.064305	0.0135	153.08	<0.0001
				β_1	-62.3728	0.3854	-161.9	<0.0001
	16.*V2*	0.001	0.932	β_0	1.381082	0.0105	131.97	<0.0001
				β_1	-59.3168	0.3014	-196.8	<0.0001
	16.*V3*	0.002	0.887	β_0	1.498366	0.0152	98.65	<0.0001
				β_1	-60.4855	0.4269	-141.7	<0.0001

RMSE = Root mean square error; R^2_{Adj}= Adjusted determination coefficient; *Ase* = Approximate standard error.

In the results of the relationships, it was observed that the equations with the best statistical adjustment exhibited inconsistencies in the trends, and therefore we performed a graphic analysis (Figure 2), in which models 4, 8 and 14 were found to be the ones that best predict the information, while 10.*V2* and 15.*V2* in combination with the expression proposed by Probosque are the ones that best adjust to the sample used, as the shape factor turned out to be high, and the expression of Vargas *et al.* (2017) yielded results below the trend. However, the fact that the equations were developed for natural forests, and the growth habit is different from that of a CFP, must be considered.

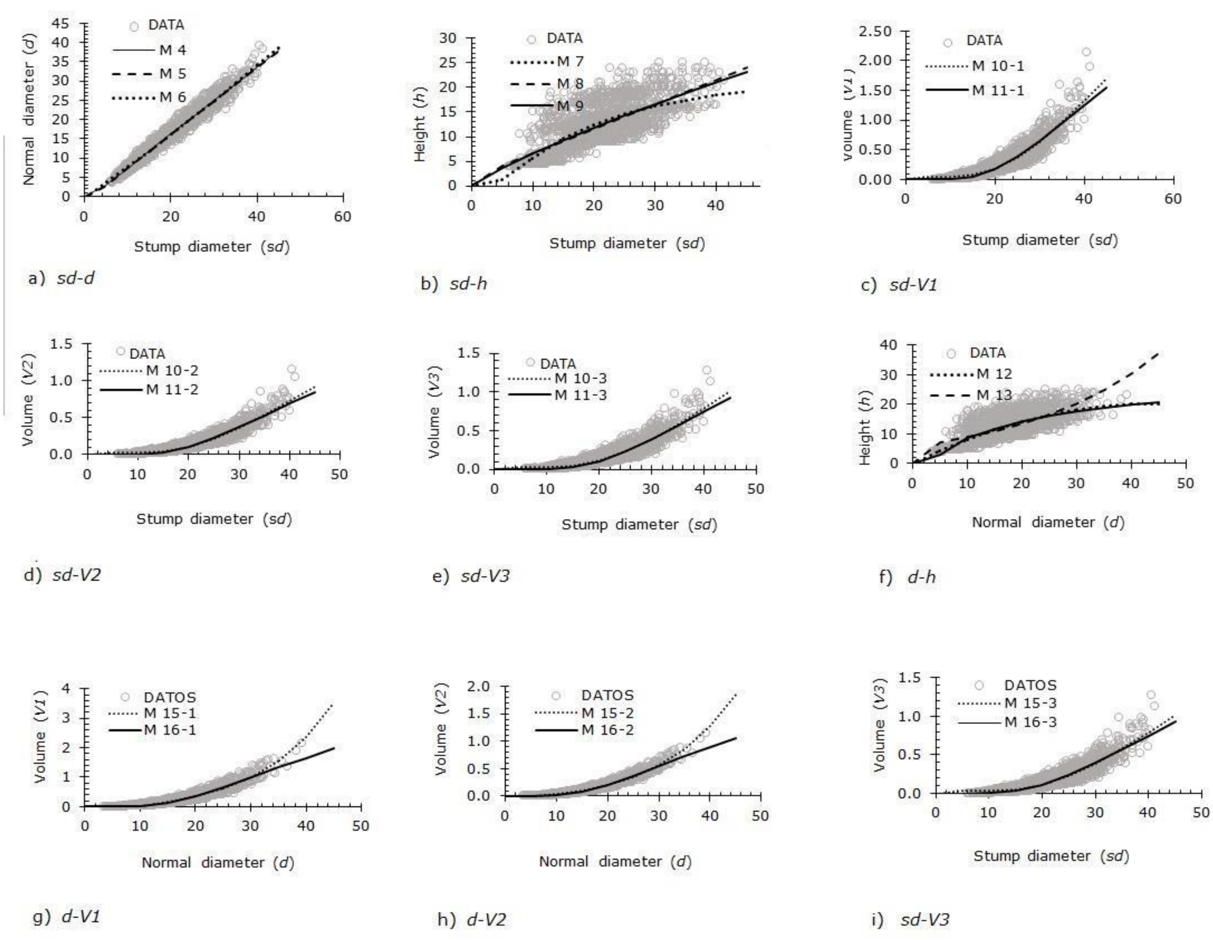


Figure 2. Prediction of the allometric relationships versus observed data of stump diameter (*sd*), normal diameter (*d*), total height (*h*), and volume (*V*) for *Pinus patula* Schiede ex Schltdl. *et* Cham. forest plantations in the State of Mexico.

In the evaluation of the estimations made using the selected model, and with the *sd* as the independent variable, the bias was 0.00001 cm, 0.000002 m and 0.00000004 m^3 for *d*, *h*, and *V*, respectively, and of 0.000011 m and 0.00252 m^3 in the estimation of *h* and *V* based on the *sd*. On the other hand, based on the *sd*, the bias was similar for models *V1* and *V3*, with 0.000001 m^3, and based on the *d*, the bias was 0.0053 m^3 for *V1*, and 0.0030 m^3 for *V2*. While, according to the statistical test, the estimations between models *V1*- *V2* and *V1*-*V3* are different ($p<0.05$), but those between models *V2*-*V3* are equal ($p=0.03$).

Discussion

The results from model 4 explain 96.0 % of *d* variability based on the dimensions of the *sd*, and, therefore, the R^2_{adj} can be considered high according to Gujarati and Porter (2010), who propose that values around 0.8 in the models are synonymous of efficiency when used. When comparing the values thus obtained, it becomes evident that they are similar to those reported by Benítez *et al.* (2004), who used a log-type model that explains more than 95% of the variability of the data by adjusting the *d*-*sd* relationship for *Casuarina equisetifolia* Forst. in *La Providencia Camagüey*, *Cuba*, besides obtaining an aggregate difference that overestimated the *d* by 1.97 cm; for their part, Bava and López (2006) used a logarithmic model for *Nothofagus pumilio* (Poepp. & Endl.) Krasser and generated a result that accounts for 97 %, and Quiñonez *et al.* (2012) utilized allometric relationships for different *Pinus* and *Quercus* species, and obtained coefficients that account for more than 92 % of the variability in the *d*-*sd* relationship. However, this percentage is lower than that reported by García *et al.* (2017), who predicted nearly 99 % of the *d* based on the *sd* and presented a RSME of 1.777 cm for *Abies religiosa* (Kunth) Schltdl. *et* Cham.

Modeling the *h* based on the *sd* and the *d* resulted in an R^2_{Adj} of 0.53 and 0.57, respectively, a situation that agrees with that described by Diéguez *et al.* (2003). These values are indicative of the complexity of modeling this radio due to the great variability and distribution of the information. An example of this are the equations proposed by Quiñonez *et al.* (2012) for *Pinus arizonica* Engelm., *P. ayacahuite* Ehrenb. ex Schltdl., *P. durangensis* Martínez, *P. leiophylla* Schiede ex Schltdl. *et* Cham., *P. teocote* Schiede ex Schltdl. *et* Cham. and *Quercus sideroxila* Bonpl., with values between 0.47 and 0.77; also, García *et al.* (2017) reported coefficients between 0.37 and 0.68 for species of commercial interest in the tropical forests of *Quintana Roo.*

For the volume, the results obtained are acceptable, as they account for 89.1 % of the variation of the data, a value that is similar to that reported by Quiñonez *et al.* (2012), who used an allometric model in their linearized form and obtained an explanation for 90 % of the variation, and results accounting for 86 % to 93 % were obtained by García *et al.* (2017), who infer that $y=a{\cdot}x^b$ is the adequate model type for predicting the volume based on the *sd*.

Model 10 is observed to be more reliable for estimating the volume (*V1, V2* and *V3*); however, among the equations, we may appreciate that *V2* and *V3* exhibit a lower variance in the data than *V1*, which is handled at convenience because it is a high *sf* (0.77), while the equation proposed by Probosque (1990) has more stability and is the one most utilized for estimating the individual volume of this species in the region.

The normality tests did not exhibit distribution problems (SW>0.93), and the residuals behaved homoscedastically after the correction, as described by Huang *et al.* (1992), Crecente *et al.* (2009) and García *et al.* (2017) (Figure 2). The biases and trends were acceptable, as the values of the deviations were lower than those obtained by Quiñonez *et al.* (2012) and García *et al.* (2017); furthermore, the trends of 12 the estimations agree with the values of Martínez and Acosta (2014) when estimating *d*, by García *et al.* (2017) when calculating *h*; Hernández *et al.* (2018) when projecting the volume, and by Díaz-Franco *et al.* (2007) when adjusting this type of allometric equations for biomass (*B*) and captured carbon (*C*).

As a practical way of exemplifying an application of these equations, it is proposed to evaluate a clandestine felling of a hectare, where an average *sd* of 40 cm was found when measuring the diameters of 260 stumps in the FP (260 trees ha^{-1}), and allometric equations resulted in an average *d* of 33.69 cm, an average *h* of 21.67 m, and average volumes of 1.3233 m^3, 0.7276 m^3 and 0.7957 m^3. Therefore, the timber yield can be estimated in 344.06 m^3, 189.19 m^3 and 206.89 m^3, with the expressions of *V1*, *V2* and *V3*, respectively. The values of *V2* based on a basic density of 0.5049 g cm^{-3} reported by Goche *et al.* (2011) for the species allowed projecting a value of 95.52 Mega-grams ha^{-1} for the extracted biomass, and of 47.76 Mega-grams ha^{-1} for $C0_2$, using the values proposed by Acosta *et al.* (2009).

Conclusions

The allometric models proposed as the best between the variables of forest interest —stump diameter (cm), normal diameter (cm), total height (m), and volume (m^3)— for the *Pinus patula* commercial forest plantations in Regions VI and VII of State of Mexico are statistically reliable and precise, and therefore they can be included in growth and yield systems for this cultivated species or for the assessment of the products obtained after legal or clandestine exploitation and, in general, in the development of forest management plans.

The evaluation of the differences between the volumes estimated with the volume equations utilized herein considers that the shape factor 0.77 used is high and differs from the other two equations; however, it is currently used in the forest management programs in the state (*V1*).

Although numerically speaking there is a difference between the volumes estimated by the equations proposed by Probosque (1990) and those calculated with the equations set forth by Vargas-Larreta *et al.* (2019), they are not statistically different; however, they do differ when the estimation is based on the shape factor. Therefore, either of these two options can be utilized until a specific option is developed for estimating the volumetric stock in the plantations.

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Conflict of interests

The authors declare no conflict of interests.

Contribution by author

Gustavo Ordaz-Ruíz, Jonathan Hernández-Ramos and J. Jesús García- Magaña: design of the study, data collection, data analysis, statistical adjustment, and drafting of the manuscript; Adrián Hernández-Ramos, Patricia Delgado-Valerio and Guadalupe G. García-Espinoza: editing, analysis and discussion of the document.

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