Estimación del diámetro normal, altura y volumen de *Pinus pseudostrobus* Lindl. en función del diámetro del tocón

Normal diameter, height and volume estimation of *Pinus pseudostrobus* Lindl. from the diameter of the stump

Edgar Alan Flores Morales¹, Oscar Alberto Aguirre Calderón¹*, Gerónimo Quiñónez Barraza², Marco Aurelio González Tagle¹ y Javier Jiménez Pérez¹

Resumen

Los rodales pueden ser afectados por diversos factores, como los fenómenos naturales o el manejo forestal, porqué modifican su estructura y el volumen maderable residual. La cuantificación del volumen removido en una intervención silvícola suele ser difícil, pero después de la aplicación de tratamientos silvícolas queda como evidencia el tocón; con base en sus dimensiones, es factible generar modelos de regresión para conocer el diámetro normal, la altura y el volumen en pie. Así, se midieron 83 árboles con un dendrómetro a fin de obtener la ecuación más confiable para determinar la relación funcional: a) diámetro normal–diámetro del tocón; b) altura total-diámetro del tocón; y c) volumen total-diámetro del tocón. Se ajustaron modelos lineales y no lineales a las relaciones establecidas. Para elegir el mejor modelo se utilizaron diferentes criterios de bondad de ajuste: el coeficiente de determinación, la raíz del error medio cuadrático y el coeficiente de variación. Con una ecuación lineal simple es posible predecir la relación diámetro normal-diámetro del tocón y con una ecuación polinomial, la relación altura total-diámetro del tocón. Para la relación volumen-diámetro del tocón, se generó un sistema de ecuaciones con efecto multiplicativo con el propósito de corregir la heterocedasticidad. Las ecuaciones dieron resultados estadísticamente confiables para predecir las variables de interés.


Abstract

Stands can be affected by several factors, such as natural phenomena or forest management because they modify their structure and residual timber volume. The quantification of the volume removed in a forestry intervention is usually difficult, but after the application of silvicultural treatments the stump keeps evident; based on its dimensions, it is feasible to generate regression models to know the normal diameter, height and standing volume. Thus, 83 trees were measured with a dendrometer in order to obtain the most reliable equation to determine the functional relationship between a) normal diameter – stump diameter; b) total height-stump diameter and c) total volume-stump diameter. Linear and nonlinear models were adjusted to the established relationships. Different criteria of goodness of fit were used to choose the best model: the coefficient of determination, the root of the mean square error and the coefficient of variation. With a simple linear equation you can predict the normal diameter-stump diameter ratio and with a polynomial equation, the total height-stump diameter ratio. For the volume-diameter ratio of the stump, a system of equations with multiplicative effect was generated in order to correct the heterocedasticity. The generated equations gave statistically reliable results to predict the variables of interest.

Key words: Heteroskedasticity, forest management, linear models, stump, *Pinus pseudostrobus* Lindl., mensuration variables.
Introduction

Timber harvesting has been greatly relevant for society due to the various benefits it has received as a result, so it is interesting to have an accurate estimate of timber stocks that allow sustainable harvesting planning (Diéguez et al., 2003), since forests are valued from an economic point of view depending on the proportion of volume they produce in the cutting cycle or planning turn (Robinson and Wood, 1994). It is estimated that in Mexico, timber production is 6.7 million m$^3$, of which the main genera of interest are: *Pinus* spp. with 4.6 million m$^3$ which is equivalent to 74.8% of the total timber production nationwide; and *Quercus* spp., with 0.7 million m$^3$, which makes up 10.8% (Semarnat, 2016).

However, forests are impacted by forest management, as happens when silvicultural treatments with cutting intensities greater than the volumetric increase of stands are applied. There are also other phenomena that lead to the loss of forest vegetation such as fires, pests and illegal logging (Sanhueza and Antonissen, 2014). Therefore, accurate information on how much existing volume there is and how much is extracted (Báes and Gra, 1990).

After the tree has been cut, the stump remains and it makes it possible to reconstruct the size of the removed trees, by means of mathematical models that predict the diameter, height and volume depending on the dimensions of the stump, which becomes useful to calculate the volumes cut or the loss due to natural phenomena (Corral et al., 2007).

To know the normal diameter, the total height and the volume of trees or groups of trees in a stand, from the stump diameter it has, among others, the following applications: (1) to estimate the volume of wood extracted in a final cut or in an improvement cutting (thinning) already made; (2) to check the results of harvesting operations after tree extraction; and (3) to assess the forestry practices carried out, reconstructing the previous composition of the mass when it has not been measured before (Bylin, 1982; Parresol, 1998).
It is notable that in recent years, the number of works has multiplied with respect to this topic, but in Mexico there is little research on the prediction of mensuration variables and volume (Quiñónez et al., 2012).

Currently, 47 species, 3 subspecies and 22 varieties of pine species are recognized for Mexico (Sánchez, 2008), of which 15 taxa are distributed in the state of Nuevo León; Pinus pseudostrobus Lindl. stands out for its commercial value, based on its wood that is considered to be of good quality and is mainly used for sawing (Estrada et al., 2014). Like other species, it has a wide distribution in the national territory (Sáenz et al., 2011).

Based on the above considerations, the following objective was planted, which consisted of adjusting mathematical models for the prediction of normal diameter, height and volume from the stump diameter for *P. pseudostrobus*, in the south of Nuevo León, Mexico.

**Materials and Methods**

**Study area**

The study area was the *Corona del Rosal ejido*, which is located in the *Galeana* municipality, southeast of the state of Nuevo León, Mexico. The *ejido* has an area of 6,646.42 ha and administratively it belongs to the influence area of the *Galeana* Rural Development District and the 1901Forest Management Unit (Umafor). It is located between 24°27'23" N and 24°32'51" N and the 99°53'54" W and the 100°01'34" W, in the physiographic province of the *Sierra Madre Oriental*. The climate belongs to a group of dry climates, of the semi-dry type, semi-dry temperate subtype with warm summer, with an average annual temperature between 12 °C and 18 °C. With the coldest month temperature between -3 °C and 18 °C and the warmest month temperature greater than 18 °C (García, 1973).
Collection and database

Through a directed sampling design, 83 trees with different diameter and height were selected, with the following characteristics: full canopy, not blunt, not scratched, without signs of pests and diseases and healthy, as well as a single shaft. With this information, a database was made with all categories of diameters and heights existing in the study area.

Each specimen was measured with a Criterion RD100® dendrometer to obtain the diameters at different heights. The first was at the minimum possible height of the stump cut; three sections were taken until reaching the normal diameter (130 cm), the first two 30 cm high and the third 70 cm. Subsequently, sections of 2 m in length were subsequently obtained to the tip of the tree; for each individual, the normal diameter, the total length of the shaft and of each section the diameters and lengths were measured. The volumes of the logs were calculated with the Smalian formula (1) and the final part with that of the cone (2) (Quiñónez et al., 2012; García, 2015):

\[
V = \frac{S_1 + S_2}{2} \times h \quad (1)
\]

\[
V = \frac{S_b \times h}{3} \quad (2)
\]

Where:

\( S_1 \) = Minor area (m\(^2\))

\( S_2 \) = Major area (m\(^2\))

\( S_b \) = Base area (m\(^2\))

\( h \) = Length (m)

\( V \) = Volume (m\(^3\))
The individual models of each section were summed to determine the total volume of the clean shaft.

**Used Models**

Once the shaft volume was obtained, the models initially recommended by Diéguez *et al.* (2003), Benítez *et al.* (2004), Corral *et al.* (2007) and Quiñónez *et al.* (2012) were adjusted (Table 1).

**Table 1.** Models used to predict normal diameter, total height and shaft volume from stump diameter.

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dn = a + b \ (dt) )</td>
<td>1</td>
</tr>
<tr>
<td>( dn = a + b \ (dt) ^2 )</td>
<td>2</td>
</tr>
<tr>
<td>( dn = a + b \ (dt) + c \ (dt) ^2 )</td>
<td>3</td>
</tr>
<tr>
<td>( \ln (dn) = a + b \ln (dt) )</td>
<td>4</td>
</tr>
<tr>
<td>( h_t = a + b \ (dt) )</td>
<td>5</td>
</tr>
<tr>
<td>( h_t = a + b \ (dt) ^2 )</td>
<td>6</td>
</tr>
<tr>
<td>( h_t = a + b \ (dt) + c \ (dt) ^2 )</td>
<td>7</td>
</tr>
<tr>
<td>( \ln (ht) = a + b \ln (dt) )</td>
<td>8</td>
</tr>
<tr>
<td>( vf = a + b \ (dtoc) )</td>
<td>9</td>
</tr>
<tr>
<td>( V_f = \beta_0 \ (\beta_1 \ (dt) ^a_2 \ )^2 (a_3 \ (dt) ^a_4 ) )</td>
<td>10</td>
</tr>
</tbody>
</table>

\( dt = \) Stump diameter; \( dn = \) Normal diameter; \( ht = \) Total height; \( vf = \) Shaft volume and \( \beta_0, \beta_1, a, b \) and \( c = \) Parameters to be estimated
The models that were used except number 10 are linear in their parameters which made the adjustment, so their adjustment was made with the method of least squares, using the MODEL procedure; and for model 10, their estimators were generated using the seemingly unrelated regression method (SUR) of the SAS statistical program (SAS, 2013).

Hair et al. (1999) and Quiñónez et al. (2012) point out that the equations that best fit the sample do not always lead to the most accurate estimates of the actual values, so a regression analysis does not consist in determining the best fit only for the sample, but developing the model that describes with greater precision the population as a whole.

The adjustment capacity was analyzed from the residuals and three statistics frequently used in the comparison of biometric models (Prodan et al., 1997; Gadow and Hui, 1999; Castedo and Álvarez, 2000; Diéguez et al., 2003; Corral et al., 2007): Root of the mean square error (3), adjusted coefficient of determination (4) and coefficient of variation (5):

\[
R_{\text{EMC}} \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \bar{Y}_i)^2}{n-p}} \quad (3)
\]

\[
R_{\text{adj}}^2 = 1 - \frac{\sum_{i=1}^{n} (Y_i - \bar{Y}_i)^2}{\sum_{i=1}^{n} (Y_i - \bar{Y}_i)^2} \left( \frac{n-1}{n-p} \right) \quad (4)
\]

\[
V_C = \sum_{i=1}^{n} \frac{(y_i - \bar{y}_i)^2}{\bar{y}} \quad (5)
\]

The root of the mean square error indicates the accuracy of the estimates; the adjusted coefficient of determination reflected the total variability explained by the model, based on the total number of parameters to be estimated, the coefficient of variation explained the relative variability with respect to the mean, which is useful for a fast comparison of the proposed models (Diéguez et al., 2003).
Likewise, the residual values were analyzed graphically against the predicted values to identify if there are outliers or some systematic tendency; this option is very practical for the analysis of fit of the aforementioned models.

Table 2 shows the statistics of stump diameter ($dt$), normal diameter ($dn$), total height ($ht$) and shaft volume ($vf$).

Table 2. Descriptive statistics of the data base.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
<th>VC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dn$ (cm)</td>
<td>45.43</td>
<td>15.82</td>
<td>14</td>
<td>66.6</td>
<td>40.67</td>
</tr>
<tr>
<td>$dt$ (cm)</td>
<td>38.89</td>
<td>17.31</td>
<td>17</td>
<td>78.8</td>
<td>38.09</td>
</tr>
<tr>
<td>$ht$ (m)</td>
<td>19.08</td>
<td>4.88</td>
<td>8.4</td>
<td>30.7</td>
<td>25.61</td>
</tr>
<tr>
<td>$vf$ (m$^3$)</td>
<td>1.65</td>
<td>1.3</td>
<td>0.079</td>
<td>6.16</td>
<td>83.96</td>
</tr>
</tbody>
</table>

$dn$ = Normal diameter (cm); $dt$ = Stump diameter (cm); $ht$ = Total height (m), $vf$ = Shaft volume (m$^3$); $SD$ = Standard deviation; $VC$ = Variation coefficient

**Results**

**Normal diameter – stump diameter relation**

The values of the model estimators for the ratio of the normal diameter as a function of the stump diameter are shown in Table 3. The ones with the highest $R^2_{adj}$ were 1, 3 and 4; however, number 3 has two non-significant parameters, which is also an indicator for the choice of the model, while for models 1 and 4 very low values were obtained in the REMC and similar ones in $R^2_{adj}$; the selected model was number 1.
Table 3. Values of the parameters and statistics of the goodness of fit for the models used for the normal diameter – stump diameter relation.

| Model | REMC   | \( R^2_{adj} \) | VC   | Parameter | Estimator  | Approximate standard error | t Value | Pr > |t| |
|-------|--------|-----------------|------|-----------|------------|-----------------------------|---------|-------|
| 1     | 2.2325 | 0.98            | 5.7  | a         | -2.21829   | 0.6919                      | -3.21   | 0.0019|
|       |        |                 |      | b         | 0.90499    | 0.0142                      | 63.54   | <0.0001|
| 2     | 3.4092 | 0.95            | 8.71 | a         | 1.598826   | 0.6719                      | 23.79   | <0.0001|
|       |        |                 |      | b         | 0.009707   | 0.000236                    | 41.05   | <0.0001|
| 3     | 2.2464 | 0.97            | 5.7  | a         | -2.1022    | 18.075                      | -1.16   | 0.2483|
|       |        |                 |      | b         | 0.899012   | 0.0871                      | 10.32   | <0.0001|
|       |        |                 |      | c         | 0.000066   | 0.000947                    | 0.07    | 0.9447|
| 4     | 0.0659 | 0.97            | 1.83 | a         | -0.41696   | 0.06355                     | -6.56   | <0.0001|
|       |        |                 |      | b         | 106.666    | 0.01691                     | 63.06   | <0.0001|

Figure 1 shows the trend of the adjustment of Model 1 that had the best statistics, in addition to the predicted values against residues obtained with the same model. The distribution of errors does not follow a defined pattern, so it is assumed that the model does not present problems of heterocedasticity; the range of residual values is small in all cases.

Figure 1. Selected model fit and graphics of the residual against the predicted values.
Total height–stump diameter relation

The values of the estimators and the statistics of fit for the total height relationship according to the stump diameter are shown in Table 4. The selected model for the species under study was number 7, which satisfactorily explained the relationship between the mensuration variables, while model 8 had the best statistics of fit, but as the stump diameter increased, the height tends to be overestimated with respect to the observed diameters.

Table 4. Values of the parameters and statistics of the goodness of fit for the models used for the total height–stump diameter relation.

| Model | REMC  | $R^2$ adj | VC  | Parameter | Estimator | Approximate standard error | t Value | Pr > |t| |
|-------|-------|-----------|-----|-----------|-----------|---------------------------|---------|------|---|
| 5     | 2.9411| 0.63      | 15.31| a        | 8.797896  | 0.9116                    | 9.65    | <0.0001 |
|       |       |           |      | b        | 0.226432  | 0.0188                    | 12.07   | <0.0001 |
| 6     | 3.0717| 0.61      | 15.99| a        | 1.342368  | 0.6054                    | 22.17   | <0.0001 |
|       |       |           |      | b        | 0.002399  | 0.000213                  | 11.26   | <0.0001 |
| 7     | 2.9454| 0.63      | 15.24| a        | 6.885309  | 2.37                      | 2.91    | 0.0047 |
|       |       |           |      | b        | 0.324927  | 0.1142                    | 2.85    | 0.0056 |
|       |       |           |      | c        | -0.00109  | 0.00124                   | -0.87   | 0.384 |
| 8     | 0.17214| 0.63    | 5.91 | a        | 0.91506   | 0.16678                   | 5.49    | <0.0001 |
|       |       |           |      | b        | 0.53487   | 0.04439                   | 12.05   | <0.0001 |

Figure 2 illustrates the trend of adjustment of model 7, which, being the best, was the chosen model; The predicted values against the residuals obtained are also shown. The distribution of errors does not follow a definite pattern, so it is assumed that the model does not present heterocedasticity problems, like number 1, for the total height-diameter of the stump ratio.
Shaft volume–stump diameter relation

The values of the estimators of the models and the adjustment statistics for the volume of the shaft according to the stump diameter are summarized in Table 5. Model 10 of the spindle volume had an $R^2_{adj}$ of 0.93 and a low REMC as well as the VC for *P. pseudostrobus*. All parameters were significant, and it was adjusted with the seemingly Unrelated Regression method (SUR), since consistent estimators are obtained for the equations that make up the system, which represents an advantage over the ordinary square minimum (MCO) (Cruz et al., 2008).
Table 5. Values of the parameters and statistics of the goodness of fit for the models used for shaft-volume – stump diameter relation.

| Model | Parameters | Estimator | Approximate standard error | REMC | $R^2_{adj}$ | Pr > |t| |
|-------|------------|-----------|-----------------------------|------|-------------|-------|---|
| 9     | a1         | -1.76885  | 0.2453                      | 0.4749 | 0.88   | <0.0001 |
|       | a2         | 0.07525   | 0.1158                      |       |       | <0.0001 |
| 10    | a1         | 0.664366  | 0.0495                      | 0.3508 | 0.93   | <0.0001 |
|       | a2         | 1.065056  | 0.0186                      |       |       | <0.0001 |
|       | a3         | 2.700923  | 0.4625                      |       |       | <0.0001 |
|       | a4         | 0.516857  | 0.0438                      |       |       | <0.0001 |
|       | $\beta_0$ | 0.000042  | 5.699E-7                    |       |       | <0.0001 |

Like García (2015), the problem of heterocedasticity associated with the shaft volume was corrected with a power function of the residual variance $\sigma_i^2 = (D^2H)^q$. The estimators of the system are significant Pr > |t| calculated with a 95 % reliability; the parameters were estimated and programmed in the SAS / ETSTM MODEL procedure (SAS, 2013) specifying:

$$resid.Vf = resid.Vf/[(D^2H)]^{0.5}.$$ 

Figure 3 describes the adjustment trend of the best fustal volume model, which suggests good statistical results, and the predicted values against the residues obtained with the model are also shown. After correcting heterocedasticity, the distribution of errors does not follow a defined pattern, so it is assumed that the model no longer has heterocedasticity problems.
Due to the simplicity of the linear model, and considering the statistics of fit, model 1 is the most suitable for estimating the normal diameter from the stump diameter for *P. pseudostrobus*, a fact that agrees with Quiñónez et al. (2012) who identified that the linear model explains the normal diameter and stump diameter relationship with an adjustment greater than 0.9, as Diéguez et al. (2003) and Corral et al. (2007) did. Model 4 has a good fit, since it has the lowest REMC and a lower VC. In a recent study, García et al. (2016) found that a simple linear equation also satisfactorily explains the relationship between the stump diameter and the normal diameter for *Abies religiosa* (Kunth) Schltdl. et Cham. in Pico de Tancítaro, in the state of Michoacán.

About the total height – stump diameter ratio there is little research. Quiñónez et al. (2012) concluded that a simple linear model had the best statistics for six pine species. In the present work the best fit was in models 5, 7 and 8 with an $R^2_{adj}$ of 0.63; when analyzing the other statistics, it was decided to choose model 7 which is polynomial; these values are well below those recorded by López and Ramos (2014).
who obtained the best fit with a potential allometric model for *Quercus laurina* Humb. & Bonpl. in *Ixtlán, Oaxaca* with an $R^2$ of 0.97.

Regarding the height-diameter ratio of the stump, Diéguez *et al.* (2003) stated that the height of the stump does not provide a significant increase over the variability explained by the models; they only recommend considering the height of the stump in the case of species with particularities at the base of the trunk.

On the other hand, Martínez (2001) related the height of dominant and codominant trees with respect to their normal diameter for *Pinus patula* Schltdl. et Cham., in the *Sierra Norte de Oaxaca*, and defined that this functional relationship can be estimated through the Schumacher Model (sigmoidal model) (Schumacher, 1939).

For the relationship between the volume of the shaft and the stump diameter, the non-linear model explains the relationship, a fact that is consistent with that proposed by Martínez and Ramos (2014) for *Quercus laurina* in *Ixtlán, Oaxaca, Mexico* and by Corral *et al.* (2007) for *Pinus cooperi* C. E. Blanco, in the *El Salto* region, *Durango*, while Quiñónez *et al.* (2012) for *Pinus arizonica* Engelm., *P. ayacahuite* Ehrenb. & Schltdl., *P. durangensis* Martínez, *P. leiophylla* Schltdl. & Cham., *P. teocote* Schltdl. & Cham. and *Quercus sideroxyla* Humb. & Bonpl. in the *ejido* in *Durango*; and Diéguez *et al.* (2003) for *Pinus pinaster* Aiton, *P. radiata* D. Don and *P. sylvestris* L. in Galicia, Spain, determined that a logarithmic model satisfactorily explains the relationship between these two variables. According to statistics, model 9 can predict reliably low volumes, which it does with a very high reliability; otherwise, for higher volumes, the model overestimates predicted values. It is common that most volume models have problems of heterocedasticity, given that at higher values of diameters and height, the variation in the volumes of the trees becomes larger (Torres and Magaña, 2001).
Conclusions

The estimation of the normal diameter from the stump diameter can be done reliably by means of a simple linear regression (1), since it reaches reliable statistics with \( R^2_{adj} > 0.90 \) and REMC of 2.2325. Height can be accurately estimated based on the stump diameter through the polynomial model (7), whose properties best described the relationship between the two variables. To estimate the volume, a non-linear model (9) was used, based on a system of equations, which reduced the heterocedasticity error, with which the volume of the shaft can be assessed more accurately.

The generated functional relationships are relevant to reconstruct scenarios in the areas affected by natural conditions or adverse effects within the study area. This information is useful for determining the individual dimensions and the total volume of \( P. \) pseudostrobus.

The selected models successfully predict the behavior of the variables normal diameter, total height and volume, respectively, depending on the stump diameter and silviculturally help to plan and make decisions about the interventions that can be applied, as well as recreate scenarios, and know in which way the treatments were applied.

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Conflict of interests

The authors declare no conflict of interests.
Contribution by author

Edgar Alan Flores Morales, Oscar Alberto Aguirre Calderón, Gerónimo Quiñónez Barraza: data analysis, model fit, writing of the manuscript; Marco Aurelio González Tagle, Javier Jiménez Pérez: review of the manuscript.

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