Article

Modelos locales altura-diámetro para *Pinus montezumae* Lamb. y *Pinus teocote* Schiede ex Schltdl. en Nanacamilpa, Tlaxcala

Local height-diameter models for *Pinus montezumae* Lamb. and *Pinus teocote* Schiede ex Schltdl. in *Nanacamilpa*, *Tlaxcala*

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Resumen

La medición en campo de la altura total de los árboles implica dificultades prácticas que incrementan los tiempos y costos del inventario. Además, su medición está propensa a errores, en particular en sitios con sotobosques densos. Las relaciones alométricas son una alternativa viable para estimar en forma eficiente la altura en función del diámetro normal. El objetivo de este estudio fue analizar modelos de crecimiento y alométricos para determinar el de mejor ajuste en la relación altura-diámetro para árboles de *Pinus montezumae* y *Pinus teocote* de la región forestal Nanacamilpa, Tlaxcala. Para el análisis, se obtuvieron 583 pares de datos de diámetro normal-altura total para *P. montezumae* y 468 pares para *P. teocote*. La muestra abarcó un amplio intervalo de categorías diamétricas. Se evaluaron 14 modelos no lineales y uno lineal, mediante análisis de regresión con el procedimiento *MODEL* de SAS® para los no lineales, y REG para el lineal. La bondad de ajuste se evaluó con base en la R^2_{adj}, el sesgo, la raíz del error cuadrático medio, la suma de cuadrado del error, el cuadrado medio del error y el criterio de información de *Akaike*. El mejor modelo para estimar la altura de *Pinus montezumae* fue el propuesto por Yang (M2) con una R^2_{adj}=0.91, y para *P. teocote* el cuadrático (M10) con una R^2_{adj}=0.87. Ambos modelos describen con precisión la relación alométrica entre el diámetro y la altura en árboles de las especies estudiadas en la región occidental del estado de Tlaxcala.

Palabras clave: Centro de México, manejo forestal, modelos de crecimiento, pinos, regresión no lineal, relaciones alométricas.

Abstract

Field measurement of total height of standing trees imposes practical difficulties that increases time and costs of forest inventories. Moreover, height measurement is prone to mistakes especially in forests with dense understories. Allometric equations are a suitable option to estimate efficiently total height as a function of tree diameter. The aim of this study was to analyze allometric models to fit the total height-normal diameter relation for *Pinus montezumae* and *P. teocote* trees in the forest region of *Nanacamilpa*, *Tlaxcala*. Database consisted of 583 pairs of total height-normal diameter measurements for *P. montezumae* and 486 pairs for *P. teocote* covering a wide range of diameter classes. We evaluated 14 nonlinear and one linear models with regression analysis using the MODEL and REG procedures of SAS® respectively. Goodness of fit was assessed with R^2_{adj}, bias, root mean squared error, sum square error, mean square error, and Akaike's information criterion. The best model for *P. montezumae* was Yang's model (M2) with an R^2_{adj}=0.91, whereas the quadratic model (M10) was the best for *P. teocote* with an R^2_{adj}=0.87. Such models described with precision the relationship between height and diameter these tree species in the western region of the state of *Tlaxcala*.

Key words: Central Mexico, forest management, growth models, pines, non-linear regression, allometric relations.

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Introduction

Forest management for timber production requires accurate measurements of the attributes or variables of the trees in order to obtain reliable estimates that may facilitate adequate decision making. Among the main variables measured in forest inventories are the total height and the normal diameter (at 1.3 m) of the individual trees, both of which are basic for estimating the volumes of timber or biomass (Xionweng and Brockway, 2017); they are also essential for studies on the growth and yield of individual trees, and therefore their usefulness is acknowledged in the execution of the technical management (Peng *et al*., 2001; Osman *et al*., 2013).

However, the in-field measurement of the total height of the standing trees, unlike that of the normal diameter, entails practical difficulties that increase the duration and cost of inventory (Trincado and Leal, 2006; Sharma, 2009). Furthermore, its measurement is liable to error, particularly in forests with dense undergrowth or where trees of different ages, species, sizes, vigors, or crown types coexist (Temesgen *et al*., 2014; Arnoni *et al*. 2016).

An alternative that overcomes the referred limitations consists in estimating the total height as a function of the normal diameter, which can be measured more easily and with greater accuracy, using mathematical models that relate the two variables (Trincado and Leal, 2006). For this purpose, linear and non-linear, mostly asymptotic models have been developed which allow a precise calculation of the total height of a tree based on terms of its normal diameter (Rijal *et al*., 2012; Arnoni *et al.,* 2016; Duan *et al*., 2018). According to Clutter *et al*. (1983), Ahmadi *et al*. (2013) and Mehtätalo *et al*. (2015), it is much better to estimate tree height with non-linear models than with linear ones.

Furthermore, it is advisable to determine the models separately for each species of interest (Misir, 2010; VanderSchaaf, 2014; Sharma and Breidenbach, 2015) and for each region (Crecente-Campo *et al*., 2010; Coble and Young-Jin, 2011). Certain studies point out that these relationships are particular and dependent on the conditions of the site conditions; therefore, it is customary to obtain local models (Corral-Rivas *et al*., 2014) with restricted application, or to incorporate into the

models certain characteristics of the site or attributes of the stand (i.e. the density, the dominant height, and the structure, among others) that also influence the height-diameter relation (Gonzalez-Benecke *et al*., 2013; Ahmadi and Alavi, 2016), which makes it possible to obtain generalized models (Corral-Rivas *et al*., 2014; Hernández *et al*., 2015) with a broader sphere of application.

Today, there are various models to adequately describe the behavior of the total height in terms of the normal diameter that have been widely used in other countries (Arnoni *et al*., 2016; Ahmadi and Alavi, 2016). For example, models like those of Chapman–Richards, Weibull, Schnute, the modified logistic model, and the exponential function have a broad application due to their predictive capacities (Peng *et al*., 2001; Temesgen *et al*., 2014; Ahmadi and Alavi, 2016).

However, despite their relevance, studies estimating the total height in terms of the normal diameter are scarce in Mexico. For example, Valdez-Lazalde and Lynch (2000) adjusted this relation using non-linear models for *Pinus patula* Schiede ex Schltdl. *et* Cham. in the state of *Puebla*; Hernández *et al*. (2015) compared 13 generalized models for analyzing the height-diameter relationship for *Pinus teocote* Schtldl. *et* Cham., while Corral-Rivas *et al*. (2014) tested 27 models and determined that the Bertalanffy-Richards growth model was the one that exhibited the best fits for six of the 13 groups of conifer and broadleaf species.

Pinus montezumae Lamb. and *P*. *teocote* are species of commercial timber importance and they are widely distributed across the country. At certain sites, the former pine species attains a height of up to 35 m and a diameter of up to 1.0 m, while *P. teocote* is a medium-sized tree with a rounded, irregular crown, which attains a height of 10 to 20 m, although the maximum values registered are 75 cm for diameter, and up to 25 m for height (Perry, 1991; Martínez, 1992). The two taxa are commercially exploited in several regions of Mexico, as in the case of the *Calpulalpan-Nanacamilpa* region, in the state of *Tlaxcala*, which has a management history of over three decades (Guerra *et al*., 2008). Due to their wide geographical distribution in the national territory, important variations in their allometry may be expected; however,

in the studied region there are no models that describe their relation of total height to normal diameter (h/d).

For this reason, the objective of the present study was to analyze 15 statistical models and to propose the model with the best fit quality in order to estimate the total height as a function of the normal diameter for *Pinus montezumae* and *P. teocote* trees in the forest region of *Nanacamilpa*, in *Tlaxcala*, Mexico.

Materials and Methods

Study area

The study region is located in western *Tlaxcala*, Mexico. To the north, it borders with the region of *Calpulalpan*, and to the south, with the state of *Puebla*; the entire region of *Nanacamilpa* is included in the physiographic region of the Transverse Neovolcanic Belt, between the geographical coordinates 19°27'30" and 19°30'38" N, and 98°38'17" and 98°32'17" W (Figure 1). It is located at an altitude range of 2 600 to 3 300 m, and its climate is of the C(w$_2$) type, i.e. temperate subhumid, with summer rains, and a mean annual temperature of 12 °C to 18 °C (Inegi, 2009).
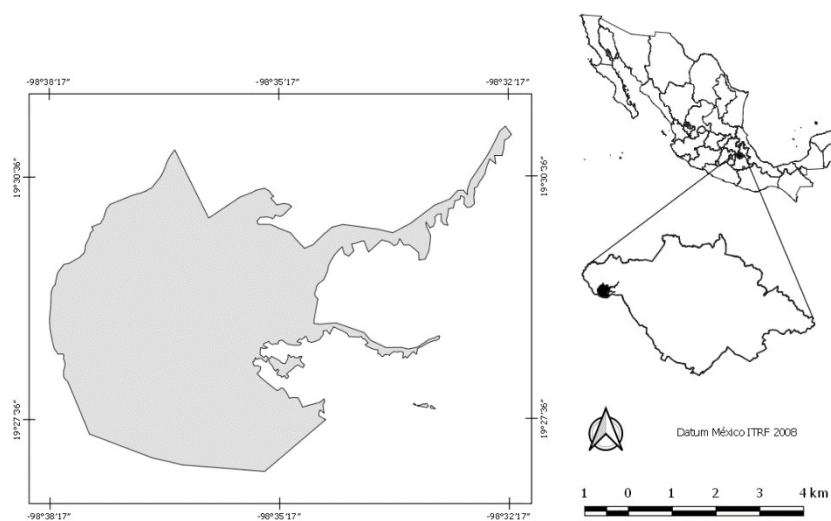

Figure 1. Location of the forest region of *Nanacamilpa, Tlaxcala*.

The most representative vegetation types correspond to the conifer and broadleaf forests, whose most prominent species are *Pinus montezumae*, *P. teocote, P. pseudostrobus* Lindl.*, Abies religiosa* (Kunth) Schltdl. & Cham. and *Quercus laurina* Bonpl., which grow on humic and eutric cambisols, as well as on humic andosols with a clayey and clayey loamy texture (Inegi, 2009). Initially, the forest management was developed under the regular system that includes regeneration fellings and thinnings during the entire rotation period (Guerra *et al.*, 2008).

Field data collection

The field information was collected by establishing sampling sites for inventory whose shape and distribution was based on the design used by the National Forest and Soils Inventory, which considers a cluster as a primary unit, made up of four 400 m² plots as secondary sampling units (Conafor, 2012). 16 systematically distributed clusters were established in the study area; in these, the dasometric variables of interest of all the living, whole and healthy trees with a normal diameter (at 1.3 m above the ground) of 7.5 or more cm were measured. Furthermore, 186 *P. teocote* trees with diameters above 30 cm —which were scarce in the sampling plots— were included.

The normal diameter was measured with a Richter® 5 m tree diameter tape, while the total height was calculated using a Haglöf Vertex III digital hypsometer with a T3 ultrasonic transmitter. Although all the tree species in each secondary sampling unit were considered, only the data for *P. montezumae* and *P. teocote* were included in the study because they were the best represented taxa in the area, having an average density of 231 and 112 trees ha^{-1}, respectively. A database for the regression analysis was built with this information, which included 583 pairs of data of normal diameter and total height for *P. montezumae* and 468 pairs for *P. teocote*. Table 1 shows the descriptive statistics.

Table 1. Basic statistics of the analyzed variables of the two studied species.

Statistic	*Pinus montezumae*		*Pinus teocote*	
	d (cm)	h (m)	d (cm)	h (m)
Minimum	7.5	2.5	7.5	4.9
Maximum	80.9	42	80.9	32.1
Average	26.7	17.73	24.9	15.7
Standard deviation	19.962	10.502	17.102	6.341
Coefficient of variation	0.75	0.59	0.69	0.40
Curtosis	-0.88	-1.32	0.13	-0.52
Assymetry	0.76	0.50	1.09	0.75

d = Normal diameter (1.30 m); *h* = Total height

Statistical analysis

The height-diameter data for *P.* montezumae and *P. teocote* were adjusted with linear and non-linear regression analyses; 14 non-linear models widely used in the specialized literature on forest biometry were tested (Table 2). Because the databases only included trees with a normal diameter of ≥7.5 cm —and therefore no diameters near or equal to zero were measured—, the constant of 1.3 was not explicitly integrated in any of the models; this constant conditions the estimated height to be 1.3 m when the normal diameter is equal to zero. Furthermore, in practical terms, only rarely is there a need to estimate heights of near-zero diameters, and therefore, their interest is merely theoretical (Trincado and Leal, 2006).

Most of the studies on the h-d relation consider non-linear models because they best describe the growth; however, in this case, a model with a linear structure in the parameters (M10) was considered, since it has sometimes proven to attain good results in the adjustment of this type of relation (García *et al*., 2017).

Table 2. Models used for determining the total height based on the normal diameter of *Pinus montezumae* Lamb. and *Pinus teocote* Schiede ex Schltdl.

Identifier	Model	Reference
(M1)	$h = \beta_0 \left[1 - e^{\beta_1 d} \right]^{\beta_2}$	Clutter *et al*. (1983)
(M2)	$h = \beta_0 \left[1 - e^{\beta_1 \, d^{\beta_2}} \right]$	Temesgen *et al*. (2014)
(M3)	$h = e^{\left(\beta_0 + \frac{\beta_1}{d + \beta_2} \right)}$	Temesgen *et al.* (2014)
(M4)	$h = \beta_0 \, e^{\left(-\beta_1 \, d^{-\beta_2} \right)}$	Castillo-Gallegos *et al.* (2018)
(M5)	$h = \dfrac{d^{\beta_0}}{\beta_1 + \beta_2 \, d^{\beta_0}}$	Castillo-Gallegos *et al*. (2018)
(M6)	$h = \dfrac{\beta_0 \, d}{\beta_1 + d}$	Castillo-Gallegos *et al.* (2018)
(M7)	$h = \beta_0 \, d^{(\beta_1 + \beta_2 \, d)}$	Sharma (2009)
(M8)	$h = \dfrac{\beta_0}{1 + \beta_1^{-1} \, d^{-\beta_2}}$	Castillo-Gallegos *et al.* (2018)
(M9)	$h = \beta_0 + \beta_1 d^{\beta_2}$	García *et al*. (2017)
(M10)	$h = \beta_0 + \beta_1 d + \beta_2 d^2$	García *et al*. (2017)
(M11)	$h = \beta_0 \, e^{\beta_1 \left(\frac{1}{d} \right)}$	García *et al*. (2017)
(M12)	$h = e^{\beta_0 + \beta_1 \, ln(d)}$	García *et al*. (2017)
(M13)	$h = \beta_0 \left(1 + \dfrac{1}{d} \right)^{-\beta_1}$	Castillo-Gallegos *et al*. (2018)
(M14)	$h = \beta_0 \left(-ln \left(\dfrac{1}{d} \right) \right)^{\beta_1}$	Castillo-Gallegos *et al*. (2018)
(M15)	$h = \left(\beta_0 + \dfrac{\beta_1}{d} \right)^{-\beta_2}$	Castillo-Gallegos *et al*. (2018)

h = Total height; d = Normal diameter; β_i = Parameters of the model.

The regression analysis was performed with the SAS® software, version 9.3 (SAS, 2011). The MODEL procedure with ordinary least squares was used for non-linear models, while the REG procedure of the same statistical analysis software was utilized for the linear model. The goodness-of-fit of each model was assessed with the coefficient of determination corrected by the number of parameters ($_{adj}R^2$) as an indicator of the proportion of the variance accounted for by the independent variable, the bias, the root of the mean squared error (RMSE), the sum of the squared error (SSE), the mean squared error (MSE), the significance of the estimators, and the Akaike information criterion (AIC). The latter parameter is a very reliable criterion, as it compares between models with different numbers of parameters (Sharma, 2009).

In order to attain greater objectivity in the selection of the best model for each species, the goodness-of-fit criteria indicated were hierarchized according to the procedure described by Tamarit-Urias *et al*. (2014), in order to generate a quality index whose total value facilitated the identification of the model with the best fit in each case. Although the statistical adjustment criteria constituted the main basis for the selection of the best model, its biological consistency was also analyzed, since it is a desirable characteristic in any growth model (Sharma, 2009). Certain authors have pointed out the importance of considering the predictive capacity of the models —particularly for the height of larger trees (Moore *et al*., 1996; Zhao *et al*., 2006)—, which entails the search for a reasonable behavior of the fitted curve through the entire data range, paying special attention to the upper limit.

Due to this, the asymptotic behavior of the curve of each species was judged in terms of the maximum values for growth in height cited in the literature and in the observed data. Thus, the graphic description of the model within and outside the range of the observed data was an additional criterion for the selection. Furthermore, the fulfillment of the basic assumptions of the regression (i.e. normality in the distribution of the errors) was verified with Shapiro-Wilk's test, along with the graphic analysis of the residuals for observing the tendencies before making inferences about the estimators of the parameters in each model (Kutner *et al*., 2005).

Results and Discussion

Pinus montezumae

Table 3 shows the adjustment of the analyzed models. In general, all the tested models for *P. montezumae*, regardless of their form and the number of their parameters, were observed to exhibit good fits, as the variance ($_{adj}R^2$) accounted for by the independent variable was above 89 % in all cases.

Table 3. Goodness-of-fit statistics of the models analyzed for *Pinus montezumae* Lamb*.*

Model	SSE	MSE	RMSE	$_{adj}R^2$	Bias	AIC	Total score
M1	5 557.8	9.58	3.10	0.91	0.03	1 320.53	(29)
M2	5 538.2	9.55	3.09	0.91	0.05	1 318.48	(28)
M3	5 582.9	9.63	3.10	0.91	0.01	1 323.16	(31)
M4	5 772.1	9.95	3.16	0.91	0.02	1 342.59	(58)
M5	5 621.4	9.69	3.11	0.91	0.04	1 327.16	(40)
M6	5 747.6	9.89	3.15	0.91	-0.06	1 338.11	(47)
M7	5 487.9	9.46	3.08	0.91	0.04	1 313.16	(21)
M8	5 621.4	9.69	3.11	0.91	0.04	1 327.16	(47)
M9	5 936.8	10.24	3.20	0.91	0.0004	1 358.99	(64)
M10	5 477.7	9.44	3.07	0.91	0.001	1 312.07	(10)
M11	6 803.6	11.71	3.42	0.89	0.20	1 436.44	(90)
M12	6 357.6	10.94	3.31	0.90	-0.09	1 396.91	(66)
M13	6 634.2	11.42	3.38	0.90	0.19	1 421.74	(84)
M14	5 858.9	10.08	3.18	0.91	-0.002	1 349.29	(58)
M15	5 656.3	9.75	3.12	0.91	0.02	1 330.77	(47)

$_{adj}R^2$ = Adjusted coefficient of determination; RMSE = Root of the mean squared error; SSE = Sum of squared error; MSE = Mean squared error; AIC = Akaike information criterion.

The fit quality index, given by the total score of each model, evidence that the five best models are M10, M7, M2, M1 and M3 (all of them with three parameters), because they exhibited the best —i.e. the lowest— values for all the goodness of fit criteria. Notably, the models with two parameters (M6, M11 and M14) exhibited poor values in the errors (SSE, MSE and RMSE), and therefore their total scores were generally less adequate than those of the models with three parameters. Besides, three of them failed to fulfill the assumption of normality in the residuals and therefore they were discarded in the selection.

Although the assessment with the index suggests which models had the best statistical behavior, the estimators of all the models were observed to be significant, except for the model with the best score (M10), which exhibited a non-significant parameter (b_0) (Table 4) and was therefore discarded.

Table 4. Values and significance of the estimators of the models analyzed for *Pinus montezumae* Lamb.

Model	b_0	Pr > |t|	b_1	Pr > |t|	b_2	Pr > |t|
M1	40.145	<.0001	0.031	<0.0001	0.144	0.0039
M2	38.681	<.0001	-0.018	<0.0001	1.135	<0.0001
M3	4.049	<.0001	-38.875	<0.0001	10.547	<0.0001
M4	113.019	<.0001	6.653	<0.0001	0.412	<0.0001
M5	1.219	<.0001	1.623	<0.0001	0.019	<0.0001
M6	72.625	<.0001	71.206	<0.0001		
M7	0.826	<.0001	1.037	<0.0001	-0.002	<0.0001
M8	51.746	<.0001	0.012	<0.0001	1.219	<0.0001
M9	-15.519	0.0009	9.941	0.0012	0.388	<0.0001
M10	0.188	0.6196	0.909	<0.0001	-0.006	<0.0001
M11	42.222	<.0001	-16.945	<0.0001		
M12	0.583	<.0001	0.716	<0.0001		
M13	42.893	<.0001	17.863	<0.0001		
M14	1.276	<.0001	2.313	<0.0001		
M15	38.664	<.0001	-0.018	<0.0001	1.135	<0.0001

b_0, b_1 and b_2 = Regression coefficients; *Pr>[t]* = Likelihood of *t*

The four next best models fulfilled the assumption of normality of residuals (H_0 is not rejected, $p < 0.05$) (Figure 2). When studying the h-d relation for 25 species in northern Mexico, Corral-Rivas *et al*. (2014) stated that the local Bertalanffy-Richards model is the most consistent, as it exhibits values of R^2 of 0.6 to 0.8 in seven pine species. The results of this work coincide with the referred study, as the model (M1), together with Yang's model (M2) —which has a similar structure— exhibited very similar results, with $_{adj}R^2$ values of 0.81. Furthermore, Castillo-Gallegos *et al.* (2018) determined that Richards' local model was the best for modeling the h-d relation for *P. chiapensis* (Martínez) Andresen in *Veracruz*, Mexico, although it had much lower R^2 values. Based on this, and having analyzed its graphic behavior, our final criterion for deciding which is the best model for *P. montezumae* was biological consistency.
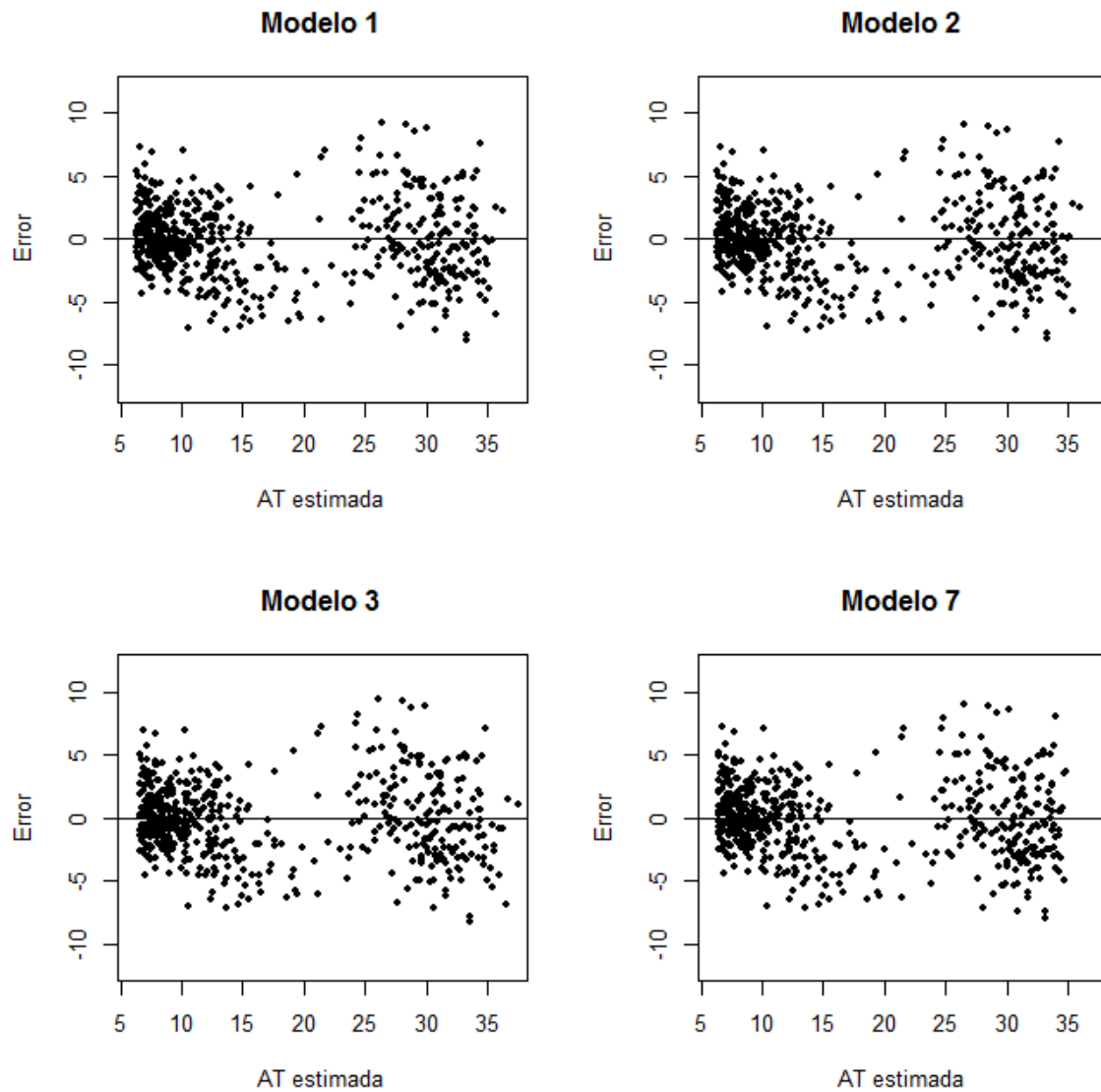
Figure 2. Distribution of residuals of the four best height-diameter models adjusted for *Pinus montezumae* Lamb.

In the observed data range, the four best models exhibited a very similar behavior, up to a diameter of approximately 60 m, after which the height projections differ (Figure 3), particularly with M3, which predicts higher values for height, without showing an asymptotic tendency even outside the data range. Conversely, Sharma's model (M7) indicates lower values and a maximum height of 35 m for diameters slightly above 80 cm, after which it predicts lower heights, which may not be altogether acceptable for *P. montezumae*.

Figure 3. Trajectories of the curves generated with the best height-diameter models and data observed for *Pinus montezumae* Lamb*.*

In this regard, Zhao *et al.* (2006) consider that two of the desirable attributes of the height-diameter equations are that they should attain an asymptote in larger diameters and that they should always exhibit a positive slope; neither attribute was observed with M7. On the other hand, the models of Richards (M1) and Yang (M2) have a similar behavior, even at the upper limit of the data range, and they exhibit an asymptotic tendency at a height of approximately 38 m, in diameters close to 100 cm. This is reasonable, from a biological point of view, for *P. montezumae* trees, particularly in regions with a good site quality (Rodríguez *et al*., 1988). However, since Yang's model (M2) is more conservative and has better adjustment values than that of Richards (M1), it is considered more adequate for estimating the total height in *P. montezumae* trees in the study region and is expressed by the following formula:

$$h = 38.681 \left[1 - e^{-0.018\, d^{1.135}} \right]$$

Where:

h = Total height

d = Normal diameter

Pinus teocote

In general, the adjustment values of the models tested for the species were also adequate, given that they are local models (Table 5). Hernández *et al*. (2015) estimated heights for *P. teocote* in the state of Hidalgo, using generalized equations, and they obtained $_{adj}R^2$ values of 0.93. According to the hierarchization of the criteria followed in this study, the quadratic model (M10) has the best fit to the data, since it has the lowest value for the index; this model's adjustment coefficient of determination ($_{adj}R^2$) is 0.87, which is above the value registered by Corral-Rivas *et al*. (2014) for the same species ($R^2<0.7$) in northern Mexico.

Table 5. Goodness-of-fit statistics of the models analyzed for *Pinus teocote* Schiede ex Schltdl.

Model	SSE	MSE	RMSE	$_{adj}R^2$	Bias	AIC	Total score
M1	2 569.5	5.53	2.35	0.86	0.01	803.00	(37)
M2	2 570.9	5.53	2.35	0.86	0.02	803.27	(51)
M3	2 518.0	5.42	2.33	0.87	0.002	793.53	(12)
M4	2 841.5	6.11	2.47	0.85	0.57	850.10	(80)
M5	2 570.9	5.53	2.35	0.86	0.01	803.26	(48)
M6	2 793.7	6.00	2.45	0.85	0.09	840.15	(72)
M7	2 568.4	5.52	2.35	0.86	0.01	802.80	(28)
M8	2 570.9	5.53	2.35	0.86	0.01	803.26	(55)
M9	2 568.5	5.52	2.35	0.86	-0.001	802.82	(25)
M10	2 505.0	5.39	2.32	0.87	0.004	791.10	(9)
M11	3 835.9	8.23	2.87	0.80	0.11	988.53	(89)
M12	2 571.1	5.52	2.35	0.86	0.003	801.29	(25)
M13	3 726.6	8.00	2.83	0.80	0.11	975.00	(83)
M14	2 733.2	5.87	2.42	0.85	0.04	829.91	(66)
M15	2 569.7	5.53	2.35	0.86	0.01	803.04	(40)

$_{adj}R^2$ = Adjusted coefficient of determination; *RMSE* = Root of the mean squared error; *SSE* = Sum of the squared error; *MSE* = Mean squared error; *AIC* = Akaike information criterion.

According to the rating system, the best models after M10 turned out to be M3, M12, M9 and M7; however, in regard to the hypothesis tests on the estimators, neither the value of b_0 with M9 nor that of b_2 with M7 were significant (P<0.05) (Table 6); therefore, they were not considered for further analysis. In any case, M3 and M12

occupied the second and third place, respectively (Table 5), with the advantage that all their coefficients were significant (p<0.05), and in both cases the assumption of normality of residuals was fulfilled, according to the dispersion graphs (Figure 4) and the SW statistic (>0.97).

Table 6. Values and significance of the estimators of the analyzed models for *Pinus teocote* Schiede ex Schltdl.

Model	b_0	Pr > t	b_1	Pr > t	b_2	Pr > t
M1	85.542	0.2886	0.002	0.6007	-0.743	<0.0001
M2	392.281	0.8367	-0.007	0.8331	0.567	<0.0001
M3	4.028	<0.0001	-63.758	<.0001	27.165	<0.0001
M4	928.458	0.6718	6.180	0.0031	0.132	0.0889
M5	0.564	<0.0001	0.363	<.0001	0.001	0.8711
M6	41.496	<0.0001	34.639	<.0001		
M7	2.651	<0.0001	0.576	<.0001	-0.0002	0.4754
M8	878.678	0.8638	0.003	0.8613	0.565	<0.0001
M9	1.197	0.5005	2.252	0.0033	0.596	<0.0001
M10	5.085	<0.0001	0.527	<.0001	-0.003	<0.0001
M11	30.357	<0.0001	-12.421	<.0001		
M12	1.030	<0.0001	0.553	<.0001		
M13	30.800	<0.0001	13.130	<.0001		
M14	2.184	<0.0001	1.768	<.0001		
M15	0.0002	0.6991	0.177	0.0017	0.572	<0.0001

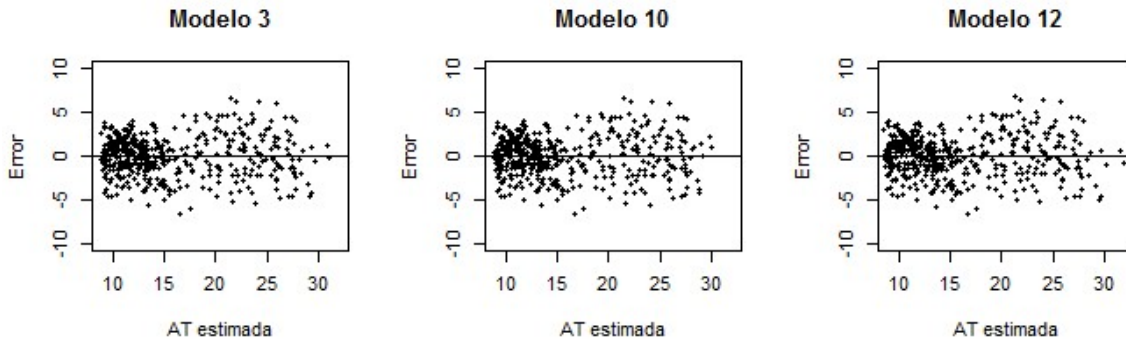b_0, b_1 and b_2 = Regression coefficients; *Pr>[t]* = Likelihood of *t*.

Figure 4. Distribution of residuals of the three best height-diameter models adjusted for *Pinus teocote* Schiede ex Schltdl.

Graphically, models M3, M10 and M12 described similar trajectories until attaining a diameter of approximately 65 cm (Figure 5), after which only model M10 exhibited a descent in the value of the slope of the curve, while M3 and M12 maintained their rising trajectory, even outside the data range. It is evident that, with these two models, there is a tendency to overestimate the total height (>32 m) for trees with diameters close to 80 cm, and therefore they are not considered adequate for modeling this relation. The tendency shown by the curve generated by model M10 seems to be more reasonable and logical for *P. teocote*, whose maximum diameters and heights can hardly exceed 85 cm and 32 m, respectively. For example, Hernández *et al*. (2015) document maximum values of 72 cm for diameter and 31 m for height in the same species.
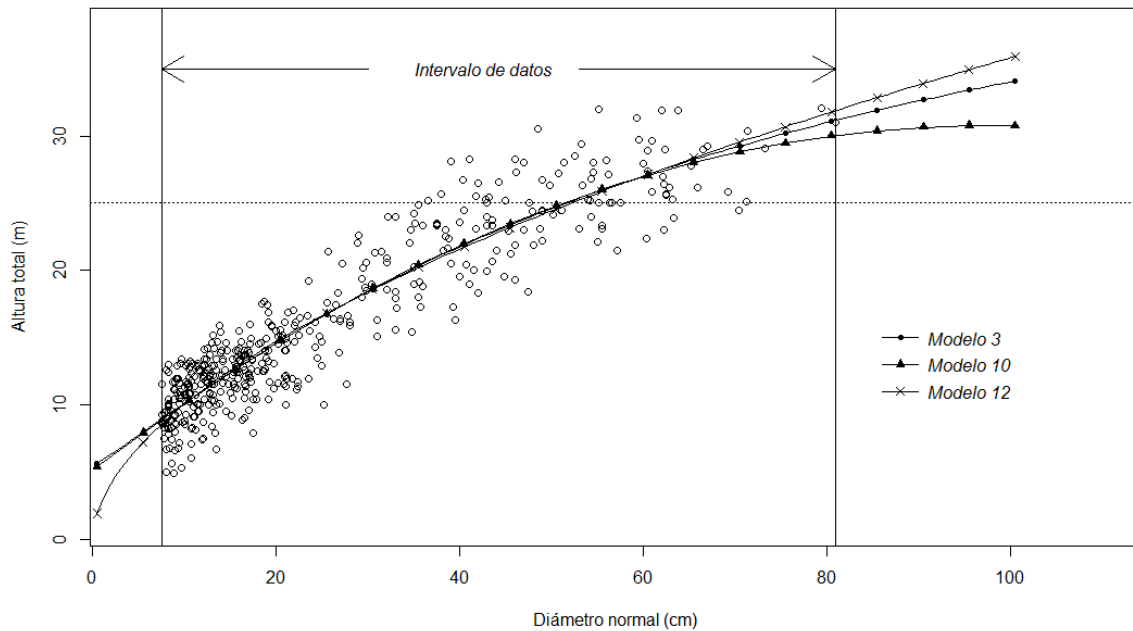
Figure 5. Trajectories of the curves generated with the best height-diameter models and data observed for *Pinus teocote* Schiede ex Schltdl.

Therefore, the final equation (Model 10) for estimating the total height of *P. teocote* trees based on the normal diameter in the study region is expressed as follows:

$$h = 5.085 + 0.527d - 0.003d^2$$

Where:

h = Total height

d = Normal diameter

The local height-diameter models selected for *Pinus montezumae* and *P. teocote* are specific to the typical conditions of the forests of the western region of the state of *Tlaxcala*. Due to their appropriate statistical adjustment values and their biological consistency, they can be applied in the forest management of this area. Furthermore,

they will be useful in future research —e.g. for generating growth and timber yield systems of the two studied *Pinus* species—, but always restricted to the same geographical area.

Conclusions

In general, the models tested for the two species have good fits, as they account for more than 80 % of the variance of the total height in both cases. The best models for estimating the total height in relation to the normal diameter for individual trees are Yang's model (M2) for *Pinus montezumae*, and the quadratic model (M10) for *P. teocote*, due to their better quality of adjustment and to the biological consistency shown in the graphic description of the height-diameter relationship. These local models will allow estimations of the total height of the trees with accuracy levels above 87 % in *P. montezumae* and *P. teocote* forests under management in the region of *Nanacamilpa*, in the state of *Tlaxcala*.

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Conflict of interests

The authors declare that they have no conflict of interests.

Contribution by author

Vidal Guerra-De la Cruz: design and structuring of the manuscript, data analysis, and drafting of the manuscript; Fabián Islas-Gutiérrez: capturing of the data, statistical analysis; Eulogio Flores-Ayala: coordination of the field work, field data collection, revision of the document; Miguel Acosta-Mireles: field data collection, analysis of the information, revision of the document; Enrique Buendía-Rodríguez: analysis of the information, drafting and revision of the document; Fernando Carrillo-Anzures: field data collection, revision of the document; Juan Carlos Tamarit-Urías: analysis of the data, revision of the document; Tomas Pineda-Ojeda: field data collection, revision of the document.

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