ON SPATIAL STATISTICS IN FORESTRY*

Matérn Bertil**

ABSTRACT

The paper tries to give historical glimpses of statistical methods related to the spatial arrangement of plants and trees in a forest. This spatial pattern has interested authors dealing with the growth of trees of stands as affected by the competition from neighbours. It has also been studied in investigations about the efficiency of operations in the forest. Most of the investigations of the spatial structure of a forest have however been connected with studies of the performance of mensurational methods and sampling designs. The angle count method is taken up as an example. Some methods that have been used for a quantitative description of the spatial pattern are reviewed. Mathematical models used to simulate spatial patterns are also taken up in recent times. The modelling has been based on the theory of stochastic processes. The French geostatistical school has developed a rich set of concepts and methods for spatial models.

RESUMEN

Este articulo comprende un breve análisis histórico de los métodos estadísticos relacionados con la distribución de plantas y árboles en un bosque. Este patrón espacial ha llamado la atención de autores interesados en el crecimiento de árboles en rodales que son afectados por la competencia. También ha sido estudiado en investigaciones relacionadas con la eficiencia de operaciones en el bosque. No obstante, la mayor parte de las investigaciones sobre la estructura espacial de un bosque, se ha asociado con estudios de comportamiento de métodos de medición y diseños de muestreo. Se toma como ejemplo el método de conteo angular. Se revisan algunos métodos utilizados para una descripción cuantitativa del patrón espacial. Recientemente se han utilizado también patrones matemáticos para simular patrones espaciales. El trazo del patrón se

* Este articulo se publica con la autorización expresa del autor.

** Dr.: Department of Forest Biometry, University of Agricultural Sciences, Umea, Sweden.

ha basado en la teoría de procesos estocásticos. La escuela francesa de Geoestadística ha desarrollado una valiosa colección de conceptos y métodos para patrones espaciales.

There are many applications of spatial statistics in forestry. This should not be surprising since geometry is involved in many concepts and theories in forestry. We may think of the spatial pattern formed by the distribution of forest land over a region, the geometrical arrangement of trees in a forest, also the geometry of the elements of the single tree. I shall here mainly deal with statistical problems and theorems connected with the *spatial arrangement of trees in a forest*.

To illustrate what this is all about, I first give a couple of examples. I then pass to a brief survey of some applications of spatial statistical methods. I must, however, warn the readers that I only give a sample, rather subjectively chosen, of theories and methods that have been applied to spatial problems in forestry.

The examples concern a very simple problem, studied by aid of a very simple spatial model. This model has been called "The Poisson forest". The term was introduced by Keuls *et al.* (1963)[1]. Trees are located completely at random in the plane. In other words, the centers of the trees form a realisation of a plane Poisson process. The properties of a tree are determined by a random experiment, independent of the location of the tree, and also independent of the location and properties of other trees. [The term "Poisson forest" has also been used as a synonym to a planar Poisson process, see Diggle (1979)[2] and Cormack (1979)[3]]. We consider now only one property: The basal area at breast height. We suppose that the basal area of individual stems has a distribution with coefficient of variation C.

Let us place **n** observation points in the plane. From each observation point we count the number of stems, which seen from the point subtend an angie > α. Let ν be the total number of trees counted in the **n** points. Denote $[100 \sin(\alpha/2)]^2$ by **k**. Then the quantity

$$\varepsilon_B = k\nu/n \qquad (1)$$

is a consistent estimate of the basal area in square metres per hectare. The angle α is often chosen so that **k** (called "the basal area factor") is 1, 2, or 4.

[1]Keuls, M.; Over, H.J. *and* De Wit. 1963. The distance method for estimating densities. pp. 71-91.
[2]Diggle, P. J. 1979. Statistical methods for spatial point patterns in ecology. pp. 95-150.
[3]Cormack, R. M. 1979. Spatial aspects of competition between individuals. pp. 151-211.

This is of course Bitterlich's famous *angle-count method*. One of the first investigations of the mathematical properties of the method was Holgate (1967)[4]. He studied its performance in a Poisson forest. Assuming that the **n** points are located in such a way that a tree in the forest can never be counted from two points, ν has a Poisson distribution with mean $\doteq$ vanance = γnk^{-1}. Here γ is the true basal area per hectare in the forest (in square meters per hectare). Then the standard error of the estimate (1) is

$$\varepsilon_B = \sqrt{(\gamma k/n)}\,. \qquad (2)$$

This can also be written

$$\varepsilon_B = \gamma / \sqrt{N_B} \qquad (2a)$$

where $N_B = E[\nu]$, i.e. the expected total number of stems counted. It should be added that expressions for ε_B in the Poisson case are also found in Grosenbaugh (1967)[5].

Let us compare this with a sample plot survey of a Poisson forest, in which **n** plots of constant area **a** hectares are chosen with the restriction that they have no point in common. Let **G** denote the basal area (in square meters) of all trees callipered in the **n** plots. The relative basal area γ is then estimated by

$$\mathbf{P = G / (na)} \qquad (3)$$

As shown by Holgate (*op. cit.*), the standard error of this estimate is

$$\varepsilon_P = \gamma \sqrt{(1 + C^2)} / \sqrt{N_P}.$$

where $\mathbf{N_P}$ is the expected total number of stems on the **n** plots. A comparison between (2a) and (4) shows that to obtain the same precision in both types of surveys, we need to measure a higher number of stems on the plots than the number of stems counted in the angle-plot survey, unless all stems have the same basal area (the case $\mathbf{C} = 0$). The formulas show that for the two surveys to give equal precision, the expected basal area of all stems *measured* in the plot survey must be the same as the expected basal area of all stems counted in the Bitterlich survey (Matérn, 1969)[6]. Using the same diameter

[4]Holgate, P. 1967. The angle-count method. pp. 615-623.
[5]Grosenbaugh, L. R. 1967. The gains from sample-tree selection with unequal probabilities. pp. 203-206.
[6]Matern, B. 1969. WiegroBist die "Relaskopfläche"?. pp. 21-22

distribution as the one assumed in Suzuki and Matsumura (1989)[7], we get C = .39. This shows that - in the Poisson case - to obtain the same precision with a sample plot survey, we must measure around 15 per cent more trees than the number counted in Bitterlich sampling. This is a low value due to the low value of C in an evenaged forest. The example illustrates the performance of different mensurational methods.

For a second example, let us consider the case **k** = 4 and ask what number of sample points that is needed in a Bitterlich sampling if we want ε_B to be as small as 2.5 % of the true value γ. This we assume to be 22.8, a value taken from an example in Suzuki and Matsumura (*op. cit.*). Using (2), we find n > 280. The same question is asked in the paper referred to, where the answer is n > 144. These authors base their computations on a model forest that is much more regular than the Poisson forest. Still the number 144 is rather large, indicating a very expensive inventory. On the other hand, if we change the example by considering a Bitterlich sampling with **k** = 1, and the more modest requirement that the ε_B shall be 5 % of the true value, formula (2) gives, still with $\gamma = 22.8$, the answer n > 17. This example illustrates the effect of the required precision on the sample size.

Holgate (*op. cit.*) also treated the case when the trees form a regular, sqare or triangular, pattern. This is mathematically more complicated than the Poisson case, but the performance of the sampling methods can still be studied by analytic methods. The angle-count method has later been studied for some more complicated patterns. Analytic solutions are given in some cases by Matérn (*op. cit.* and 1972)[8], and - in more general and advanced cases - by Penttinen (1988)[9]. Simulations will however be necessary in many cases, especially if the model is a more or less true copy of an actual forest. For such simulations, see e.g. Palley and O'Regan (1961)[10], Sukwong *et al.* (1971)[11], and the paper already referred to by Suzuki and Matsumura. It seems that the relative advantage of the relascope method over sample plots is as large or greater than in the Poisson forest.

The relascope examples illustrate what appears to be the most common use of spatial statistics in forestry: *To study statistical problems, especially the performance of mensurational methods and sampling strategies.*

[7]Suzuki, T. and Matsumura, N. 1989. Simulation der Winkezälhlprobe nach Bitterlich. pp. 998-1002.

[8]Matérn, B. 1972. The precision of basal area estimates. pp. 123-125.

[9]Penttinen, A. 1988. A random field approach to Bitterlich sampling. pp. 259-268.

[10]Palley, M. N. and O'Regan, W. G. 1961. A computer technique for the study of forest sampling methods. I. Point sampling compared with line sampling. pp.282-294.

[11]Sukwong, S., Frayer, W. E. and Mogren, E. W. 1971. Generalized compansons of the precision of fixed-radius and variable-radius plots for basal area estimates. pp. 263.271.

The earliest approach to the use of spatial statistical methods in forestry for problems of this type appeared in connection with the somewhat complicated question of the *precision of a systematic sample*, such as a regularly spaced line or sample plot inventory. The first statistical discussion of this problem appeared in the Scandinavian countries some 80 years ago when the first large scale forest inventories were planned (in Finland, Norway, and Sweden). They were based on a simple model of independent observations, a model which is analogous to the Poisson forest. Then followed models of a non-stochastic trend plus a purely random component. These models are rather unrealistic.

Attempts to a more sophisticated and more realistic model can be traced back to Langsaeter's work (1926, 1927)[12], attempts that can be said to be analogous to those that were based on the theory of *autocorrelated stochastic processes* and appeared much later in the statistical literature (Cochran, Madow, Yates and others). Such models have later been used in studies of many statistical problems in forest surveys: Comparisons of different sampling schemes, the choice of shape and size of sampling units, etc. A recent work in the long Nordic tradition in this field is Korhonen and Maltamo (1991)[13].

Another field where the pattern of trees in a forest has been of importance is the study of growth and *yield of individual trees and stands*. One distinguishes between *regular patterns*, that can be said to be in between the completely regular networks and the Poisson process, and *clustered patterns* that are more aggregated than the Poisson process. There are many factors that can produce clustering, whereas competition between plants can have a tendency to produce regularity, as does of course also artificial regeneration. It would lead too far to try to mention all the types of models that have been suggested. One model of clustering that has been applied in many fields, also in forestry, is the *center satellite model*, extensively investigated by W.G. Warren, see e.g. Warren (1971)[14]. Models involving inhibition have been applied to show the effect of competition. The geometry of the ground area occupied by a tree, or available to the tree, has been dealt with by i.a. Brown (1965)[15], Adlard (1974)[16], and Gates and Westcott (1978)[17]. Different expressions for the effect of competition on the

---

[12] Langsaeter, A. 1926-1927. Über die Berechnung des Mittelfehlers des Resultates einer regelmässigen Linientaxierung. pp. 71-89.

[13] Korhonen, K. T. and Maltamo, M. 1991. The evaluation of forest inventory designs using correlation functions. pp. 77-83.

[14] Warren, W. G. 1971. The center-satellite concept as a basis for ecological sampling. pp 87-116.

[15] Brown, G. S. 1965. Point density in stems per acre.

[16] Adlard, P. G. 1974. Development of an empirical competition model for individual trees within a stand. pp. 22-37.

[17] Gates, D. J. *and* Westcott, M. 1978. Spatial competition in plantations. pp. 98-103.

survival and growth of a tree can be studied in e.g. Newnham and Smith (1964)[18], Mitchell (1969)[19], Lundell (1973)[20], Hegyi (1974)[21], and Diggle *et al.* (1976)[22]. Surveys of the development of the subject can be found in e.g. Munro (1964)[23], and the recent investigations Pukkala (1989)[24], and Tham (1989)[25].

The modelling of the pattern is one side of the problem. Another side is the *statistical method used to express the properties of an observed pattern*. Some methods require a knowledge of the location of all trees, i.e. the case of *mapped data*, where as other methods are intended for *samples* of observation points or sample plots (usually called quadrats in ecology). Many methods are founded on so called distance methods: Distances from sample point to the nearest trees, and from a tree to its neighbours, supply information about the clustering of the tendency towards regularity, see Diggle *et al.* (*op. cit.*), and Diggle (1983)[26]. The first application of such methods to estimate the density of plants was published by the german forester Bauersachs (1942)[27]. Distance methods were further developed by ecologists in the 1950's. There is also a line of research on these methods reported in the forestry literature, see e.g. works by Essed (1954)[28], Strand (1954)[29], Stoffels (1955)[30] and Persson (1964)[31].

Investigations of the performance of the distance methods were among the first extensive applications of *simulation methods*. Some early references are Clark and Evans (1954)[32], and Cottam and Curtis (1956)[33]. For recent developments, see Diggle (*op. cit.*). The intricate network of influences from tree to tree invalidates many

[18]Newnham, R. M. & Smith, J. H. G. 1964. Development and and testing of stand models for Douglas fir and lodgepole pine. pp. 494-502.

[19]Mitchell, K. J. 1969. Simulation of the growth of even-aged stands of white spruce.

[20]Lundell, S. 1973. A model for simulating volume increment in theoretical stands.

[21]Hegyi, F. 1974. A simulation model for managing jack-pine stands. pp. 74-90.

[22]Diggle, P.; J. Besag, J. and Gleaves, J. T. 1976. Statistical analysis of spatial point patterns by means of distance methods. pp. 659-667.

[23]Munro, D. D. 1974. Forest growth models a prognosis. pp. 7-21.

[24]Pukkala, T. 1989. Methods to describe the competition process in a tree stand. pp. 187-202.

[25]Tham, A. 1989. Prediction of individual tree growth in managed stands of mixed *Picea abies* (L.) Karst., *Betula pendula* Roth and *Betula pubescens* Ehrh. pp. 491-512.

[26]Diggel, P. J. 1983. Statical Analysis of Spatial Patterns.

[27]Bauersachs, E. 1942. Bestandesmassenaufnahme nach dem Mittelstammverfahren des zweitkleinsten stammabstandes. pp. 182-186.

[28]Essed, F. E. 1954. A quick, simple and at tha same time accurate method for estimating the total volume and the increment percent of evenaged stands. pp. 260-263.

[29]Strand, L. 1954. A measure of the distribution of individuals over a certain area. pp. 191-207.

[30]Stoffels, A. 1955. Die Genauigkeit der Bestimmung der Stammzahl pro hektar durch Messung von Sstammabständen. pp. 211-218.

[31]Persson, O. 1964. Distance methods.

[32]Clark, P. J. and Evans, F. C. 1954. Distance to nearest neighbor as a measure of spatial relationships. pp. 445-453.

[33]Cottam, G. and Curtis, J. T. 1956. The use of distance measures in phytosociological sampling. pp. 451-460.

classical statistical procedures, as pointed out by Ford & Diggle (1981)[34]. Tomppo (1986)[35] surveys both models and statistical methods for analysing spatial pattern of trees.

One other application of spatial statistics that ought to be mentioned is the investigation of how operations in a forest are affected by the spatial pattern. I give two references: Newnham (1965)[36], and Almquist (1973)[37] Newnham is one of the pioneers in the field of spatial models and computer simulation in forestry, see also Newnham (1967)[38].

The French geostatistical school founded by G. Matheron has given very important contributions to the study of spatial patterns. This school has developed a wealth of concepts and methods for modelling and analysing spatial phenomena. A short survey is Miller *et al.* (1972)[39]. Applications to forestry are found in the theses Marbeau (1976)[40], and Houillier (1986)[41], to ecology in general in Rossi *et al.* (1992)[42]. Isaacs and Srivastava (1989)[43] is a recent textbook.

The construction of concepts and methods for the study of spatial patterns in forestry has perhaps an appearance of being home-made in the early applications. It has in recent times, for better or worse, been influenced by the mathematical and statistical theories of *stochastic geometry* and *spatial statistics*. These theories seem at present to be in a state of rapid development. Comprehensive textbooks did not apper until in the 1980's: Ripley (1981)[44], Diggie (*op. cit.*) y Stoyan *et al.* (1987)[45].

Spatial statistics now compnses several special applications such as *image analysis* and *stereology*. These theories deal with procedures of making inference about three-dimensional phenomena from observations in two or lower dimensions. We may include also inferences about two-dimensional surfaces from observations in one or zero dimension: Lines and points. The lines and transects of forestry and ecology are

[34] Ford, E. D. & Diggle, P. J. 1981. Competition for light in a piant monoculture modelled as a spatial stochastic process. pp. 481-500.
[35] Tomppo, E. 1986. Models and methods for analysing spatial patterns of tree.
[36] Newnham, R. M. 1965. The effect of the spatial distribution of trees on the design of harvesting machinery.
[37] Almquist, A. 1973. Simulation of logging machines.
[38] Newnham, R. M. 1967. The use of simulation models in forest research. pp. 224-254.
[39] Miller, C.; Poisonnet, M. and Serra, J. 1972. Morphologie mathematique et sylviculture.
[40] Marbeau, J. P. 1976. Géostatistique forestière.
[41] Houillier, F. 1986. Échantillonage et modelisation de la dynamique des peuplements forestiers.
[42] Rossi, R. E., Mulla, D. J., Journel, A. G. & Franz, E. H. 1992. Geostatistical tools for modelling and interpreting ecological spatial dependence. pp. 277-314.
[43] Isaaks, E. H. and Srivastava, R. M. 1989. An introduction to applied geostatistics.
[44] Ripley, B. D. 1981 Spatial statistics.
[45] Stoyan, D.; Kendall, W. S. and Mecke, 1987. Stochastic Geometry and its applications.

examples of practical applications of the stereological theory, and many new possible applications appear in the handling of information from satellites. A good example of a stereologic procedure is the angle count method of Bitterlich, and further developments of the method by Strand, Hirata and others, see Bitterlich (1984).[46]

I started this paper with a study of this method, and I may as well stop here. The relascope is the most spectacular application of spatial statistics in forestry that has appeared in the last 100 years.

REFERENCES

ADLARD, P. G. 1974. Development of an empirical competition model for individual trees within a stand. *In*: FRIES, J. (1974): 22-37.

ALMQUIST, A. 1973. Simulation of logging machines. (In Swedish. English summary). Forskningsstifteisen Skogsarbeten, Meddelande 9.

BAUERSACHS, E. 1942. Bestandesmassenaufnahme nach dem Mittelstammverfahren des zweitkleinsten Stammabstandes. Forstwissenschaftliches Centralblatt 64: 182-186.

BITTERLICH, W. 1984. The relascope idea. Commonwealth Agricultural Bureaux.

BROWN, G. S. 1965. Point density in stems per acre. New Zealand Forestry Research Notes 38.

CLARK, P. J. and EVANS, F. C. 1954. Distance to nearest neighbor as a measure of spatial relationships. Ecology 35: 445-453.

CORMACK, R. M., 1979. Spatial aspects of competition between individuals. *In*: CORMACK and ORD (1979), pp. 151-211

CORMACK, R. M. and ORD, J. K. (eds.). 1979. Spatial and temporal analysis in ecology. Statistical Ecology Series, Volume 8. International Co-operative Publishing House. Fairland, Maryland, USA.

[46] Bitterlich, W. 1984. The relascope idea.

COTTAM, G. and CURTIS, J. T. 1956. The use of distance measures in phytosociological sampling. Ecology 37: 451-460.

DIGGLE, P. J. 1979. Statistical methods for spatial point patterns in ecology. *In*: CORMACK and ORD (1979), pp. 95-150.

DIGGLE, P. J. 1983. Statistical Analysis of Spatial Patterns. Academic Press, London.

DIGGLE, P. J.; BESAG, J. and GLEAVES, J. T. 1976. Statistical analysis of spatial point patterns by means of distance methods. Biometrics 32: 659-667.

ESSED, F. E. 1954. A quick, simple and at the same time accurate method for estimating the total volume and the increment percent of evenaged stands. Paper for the IV World Foresfry Congress, and in Indian Forester 82: 260-263 (1956).

FORD, E. D. and DIGGLE, P. J. 1981. Competition for light in a plant monoculture modelled as a spatial stochastic process. Annals of Botany 48: 481-500.

FRIES, J., (ed.). 1974. Growth models for tree and stand simulation. IUFRO S4.01-4. Proceedings of meetings in 1973. Royal College of Forestry, Stockholm. Department of Forest Yield Research. Research Note 30.

GATES, D. J. and WESTCOTT, M. 1978. Spatial competition in plantations. Supplement to Advances in Applied Probability 10: 98-103.

GROSENBAUGH, L. R. 1967. The gains from sample-tree selection with unequal probabilities. Journal of Forestry 65: 203-206.

HEGYI, F. 1974. A simulation model for managing jack-pine stands. *In*: FRIES, J., 1974, pp. 74-90.

HOLGATE, P. 1967. The angle-count method. Biometrika 54: 615-623.

HOUILLIER, F. 1986. Échantillonage et modelisation de la dynamique des peuplements forestiers. Thèse présentée devant l'Université de Claude Bernard. Lyon.

ISAAKS, E. H. and SRIVASTAVA, R. M. 1989. An introduction to applied geostatistics. Oxford University Press.

KEULS, M.; OVER, H. J. & De WIT. 1963. The distance method for estimating densities. Statistica Neerlandica 17: 71-91.

KORHONEN, K. T. & MALTAMO, M. 1991. The evaluation of forest inventory designs using correlation functions. Silva Fennica 25 (2): 77-83

LANGSAETER, A. 1926, 1927. Über die Berechnung des Mittelfehlers des Resultates einer regelmässigen Linientaxierung. (In Norwegian, German summary) Meddelelser fra def norske skogforsoksvesen 2 (7): 5-47 and 2 (8): 71-89.

LUNDELL, S. 1973. A model for simulating volume increment in theoretical stands. (In Swedish, English summary). Royal College of Forestry, Stockholm, Department of Silviculture, Research Notes 2.

MARBEAU, J. P. 1976. Géostatistique forestière. Thèse présenté à l'École Nationale Superieure des Mines à Paris.

MATÉRN, B. 1969. WiegroBist die "Relaskopfläche"? Allgemeine Forstzeitung, Folge 2 (80): 21-22.

MATÉRN, B. 1972. The precision of basal area estimates. Forest Science, 18 (2): 123-125.

MILLIER, C., POISONNET, M. and SERRA, J. 1972. Morphologie mathématique et sylviculture. *In*: I.U.F.R.O., 3rd Conference, Advisory Group of Forest Statisticians, Jouy-en-Josas, 7-11 septembre 1970. Institut Nationale de la Recherche Agronomique.

MITCHELL, K. J. 1969. Simulation of the growth of even-aged stands of white spruce. Yale University, School of Forestry, Bulletin 75.

MUNRO, D. D. 1974. Forest growth models a prognosis. *In*: FRIES (1974), pp. 7-21.

NEWNHAM, R. M. 1965. The effect of the spatial distribution of trees on the design of harvesting machinery. Canada, Dept. of Forestry 65-H.5.

NEWNHAM, R. M. 1967. The use of simulation models in forest research. International Advisory Group of Forest Statisticians. Second Conference.

Stockholm, 27th September - 1st October 1965. Royal College of Forestry, Stockholm, Department of Forest Biometry, Research Notes 9: 244-254.

NEWNHAM, R.M. and SMITH, J.H.G. 1964. Development and testing of stand models for Douglas fir and lodgepole pine. The Forestry Chronicle 40: 494-502.

PALLEY, M. N. and O'REGAN, W.G. 1961. A computer technique for the study of forest sampling methods. I. Point sampling compared with line sampling. Forest Science 7: 282-294.

PENTTINEN, A. 1988. A random field approach to Bitterlich sampling. Annales academiae fennicae. Ser A1 (13): 259-268.

PERSSON, O. 1964. Distance methods. Studia forestalia suecica 15.

PUKKALA, T. 1989. Methods to describe the competition process in a tree stand. Scandinavian Journal of Forest Research 4: 187-202.

RIPLEY, B. D. 1981. Spatial Statistics. Wiley.

ROSSI, R. E.; MULLA, D. J.; JOURNEL, A. G. and FRANZ, E. H. 1992. Geostatistical tools for modelling and interpreting ecological spatial dependence. Ecological Monographs 62: 277-314.

STOFFELS, A. 1955. Die Genauigkeit der Bestimmung der Stammzahl pro Hektar durch Messung von Stammabständen. Forsfwissenschaftliches Centralblatt 74: 211-218.

STOYAN, D.; KENDALL, W.S. and MECKE, J. 1987. Stochastic Geometry and Its applications. Wiley.

STRAND, L. 1954. A measure of the distribution of individuals over a certain area. (In Norwegian. English summary). Meddelelser fra det Norske skogforsoksvesen 12: 191-207.

SUKWONG, S.; FRAYER, W. E. and MOGREN, E. W. 1971. Generalized comparisons of the precision of fixed-radius and variable-radius plots for basal area estimates. Forest Science 17: 263-271.

SUZUKI, T. and MATSUMURA, N. 1989. Simulation der Winkezählprobe nach Bitterlich. Mitteilungen der Abteilung für Forstliche Biometrie, Freiburg i. Br. 89 (2) 998-1002.

THAM, A. 1989. Prediction of individual tree growth in managed stands of mixed *Picea abies* (L.) Karst. and *Betula pendula* Roth and *Betula pubescens* Ehrh. Scandinavian Journal of Forest Research 4: 491-512.

TOMPPO, E. 1986. Models and methods for analysing spatial patterns of trees. Communicationes Instituti Forestalis Fenniae 138.

WARREN, W.G., 1971. The center-satellite concept as a basis for ecological sampling. *In*: PATIL, G.P.; PIELOU, E.C. and WATERS, W.E. (eds). Statistical Ecology 2, Pennsylvania State University Press, pp. 87-116.