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Research article

Ecuaciones de volumen fustal-total y ahusamiento para especies maderables del ecosistema templado en Puebla, México

Total-stem volume and taper equations for commercial species in the temperate ecosystem of the state of *Puebla*, Mexico

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Abstract

In forestry, accurate estimation of timber volume is essential for forest management in temperate forests. The objective of the present research was to generate equations of the total tree volume (*Vtt*), stem volume (*Vs*), and stem profile for *Pinus ayacahuite*, *Pinus leiophylla*, *Pinus hartwegii*, *Pinus montezumae*, *Pinus patula*, *Pinus pseudostrobus*, *Pinus teocote*, *Abies religiosa* and *Quercus* sp. in productive landscapes of a temperate ecosystem in the state of *Puebla*, Mexico. A sample of 1 676 trees was collected by destructive sampling. Based on the dendrometric type, the *Vs*, *Vtt*, and crown volume (*Vc*) were obtained. The volume of branches and twigs with diameters less than 5 cm was estimated using a xylometer and the Berkhout nonlinear model. The Schumacher-Hall model was fitted for the *Vtt* and the *Vs* with weighted regression in order to correct for heteroscedasticity. The Biging function was used to describe the stem profile, fitted with nonlinear generalized least squares to correct for autocorrelation. The goodness of fit of the equations are above 96 %, and the bias is below 0.04 m^3. The statistical results show that the differential equation system predicts with certainty the *Vtt*, *Vs*, as well as the taper of commercial species in the temperate forests of *Puebla*, Mexico.

Key words: Taper, allometric equations, stem profile, *Pinus* spp., stem volume, total tree volume.

Resumen

En silvicultura, la estimación precisa del volumen maderable es fundamental para el manejo forestal en bosques templados. El objetivo de la presente investigación fue generar ecuaciones de volumen total árbol (*Vta*), volumen fustal (*Vf*) y perfil del fuste para *Pinus ayacahuite*, *Pinus leiophylla*, *Pinus hartwegii*, *Pinus montezumae*, *Pinus patula*, *Pinus pseudostrobus*, *Pinus teocote*, *Abies religiosa* y *Quercus* sp. en paisajes productivos de un ecosistema templado en Puebla, México. Mediante un muestreo destructivo se recolectó una muestra de 1 676 árboles. Con base en el tipo dendrométrico se obtuvo el *Vf*, *Vta* y volumen de copa (*Vc*). Con un xilómetro y el modelo no lineal de *Berkhout* se estimó el volumen de ramas y ramillas con diámetros menores a 5 cm. Para *Vta* y *Vf* se ajustó el modelo *Schumacher-Hall* con regresión ponderada para corregir la heterocedasticidad. La función de *Biging* se usó para describir el perfil fustal, ajustado con mínimos cuadrados generalizados no lineales para corregir la autocorrelación. La bondad de ajuste de las ecuaciones fue superior a 96 % y un sesgo menor a 0.04 m^3. Los resultados estadísticos muestran que el sistema de ecuaciones diferenciado predice con certidumbre el *Vta*, *Vf* y ahusamiento de las especies maderables comerciales en los bosques templados de Puebla, México.

Palabras clave: Ahusamiento, ecuaciones alométricas, perfil fustal, *Pinus* spp., volumen fustal, volumen total árbol.

Introduction

Forest or total volumes are generally estimated based on linear and nonlinear functions. The allometric equations cited in the literature include the following: Combined variable, Korsun, Schumacher-Hall, Australian, Meyer modified, Naslund, Takata, Understandable, Logarithmic, Thornber, Berkhout, Honer, and Wenk (Clutter *et al.*, 1983; Romahn *et al.*, 1994; Prodan *et al.*, 1997). Because of its accuracy and simplicity, the Schumacher-Hall model is one of the most widely used ones (Corral-Rivas and Návar-Cháidez, 2009; Vargas-Larreta *et al.*, 2018).

According to Burkhart and Tomé (2012), there are several equations for describing a tree's stem profile. The most commonly used ones are Bruce *et al.* (1968), Demaerschalk (1972), Ormerod (1973), Max and Burkhart (1976), Cao *et al.* (1980), Clutter (1980), Biging (1984), Kozak (1998), Rentería-Anima and Ramírez-Maldonado (1998), Bi (2000), Fang *et al.* (2000), Sharma and Oderwald (2001), among others.

During the last two decades, in Mexico, adjustments of taper functions and commercial volume have been generated for several tree species of the temperate ecosystem: for *Abies religiosa* (Kunth) Schltdl. & Cham. in various regions of the country (Guzmán-Santiago *et al.*, 2022), for *Pinus greggii* Engelm. *ex* Parl. in the state of *Hidalgo* (Hernández-Ramos *et al.*, 2017), for *Quercus sideroxyla* Bonpl. in the state of *Durango* (Quiñonez-Barraza *et al.*, 2019), for *Quercus* sp. in the state of *Puebla* (Tamarit *et al*., 2017); for *Pinus pseudostrobus* Lindl. in the state of *Nuevo León* (Flores *et al.*, 2021), for *Pinus oocarpa* Schiede *ex* Schltdl., and for *Pinus douglasiana* Martínez in *Durango* (López *et al.*, 2015); for *Pinus teocote* Schltdl. & Cham. in *Nuevo Léon* (Tapia and Návar, 1998), and for *Pinus cooperi* C. E. Blanco, *Pinus durangensis* Martínez, *Pinus engelmannii* Carrière, *Pinus leiophylla* Schiede *ex* Schltdl. & Cham., *Pinus herrerae* Martínez, *Pinus ayacahuite* C. Ehrenb. *ex* Schltdl., *Pinus arizonica* Engelm., and *Pinus teocote* in *Durang*o (Corral *et al*., 1999; Quiñonez-Barraza *et al.*, 2014); for *P. ayacahuite* in the state of *Oaxaca* (Ramírez-Martínez *et al.*, 2018) and for *P. cooperi* in *Durango* (Cruz-Cobos *et al.*, 2008), among others.

The obtained results with Bigin's (Biging, 1984) and Fang's equation (Fang *et al.*, 2000) have been very accurate in describing the stem profile (Corral-Rivas and Návar-Cháidez, 2009; Pompa *et al.*, 2009). Biging's equation (Biging, 1984) has the advantage of presenting only two parameters, in contrast with other functions.

Understanding and quantifying wood volume is essential to planning and implementing sustainable forest management (Ramírez-Martínez *et al.*, 2018; Flores *et al.*, 2021). Tree volume can be estimated in parts or as a whole, including stem, branches, and twigs (Vargas-Larreta *et al.*, 2018). The stem volume refers to the volume of the straight and cylindrical part of the trunk, or it may be limited to a certain portion of the stem. The latter is determined using taper functions, which predict the profile of the tree trunks at different heights.

In 2013, the *Comisión Nacional Forestal* (National Forest Commission) and several institutions in Mexico generated the project: "Development of a biometric system for forest management planning in ecosystems with timber potential in Mexico" which was developed for the state of *Chihuahua*, *Guerrero*, *Jalisco*, *Durango*, *Oaxaca*, *Michoacán*, *Puebla*, *Estado de México*, *Hidalgo*, *Tlaxcala*, *Veracruz and Quintana Roo*; 6 414 equations in it were generated to estimate volume, taper and site quality (Vargas-Larreta *et al.*, 2018). The Schumacher and Hall model (Schumacher, 1933) was used to calculate the timber volume, and Fang's compatible model was used for the taper and commercial volumes. For the crown volume, when estimating the total tree volume, this biometric system considered only the volume of branches and twigs with diameters above 5 cm, and did not generate the tree and total tree equations separately; thus, although these estimates are covered by additive equations, in their practical application they correspond to the marketing of the tree; while in the traditional technical management applied in *Puebla*, the differentiated equations are of greater use and operational importance and also take into account the volume of branches and twigs with diameters of less than 5 cm.

According to Santos (2023), the total stem volume functions by species group, generated as part of the National Forest Inventory more than 40 years ago, are still in use in the state of *Puebla*. Therefore, the objective was to generate differentiated and simplified equations for the total tree volume, the stem volume, and the stem profile for *Pinus ayacahuite*, *P. hartwegii* Lindl., *P. leiophylla*, *P. montezumae* Lamb., *P. patula* Schltdl. & Cham., *P. pseudostrobus*, *P. teocote*, *Abies religiosa* and *Quercus* sp., which are the main commercial timber species in the temperate ecosystem of *Puebla*, Mexico.

Materials and Methods

Study area

The study was conducted in forest areas with timber harvesting in *Puebla*, Mexico, specifically in the Forest Management Units (*Umafor,* by its acronym in Spanish) No. 2101, *Izta Popo* (642 676 ha); No. 2103, *Teziutlán* (324 329 ha); No. 2105, *Centro y Pico de Orizaba* (414 817 ha), and No. 2108, *Chignahuapán-Zacatlán* (271 853 ha) (Conafor, 2023). The Umafors are located between the coordinates 97°10' and 98°45' W and 18°10' and 20°15' N (Figure 1). The species of timber forest importance for *Puebla —Pinus ayacahuite*, *P. hartwegii*, *P. leiophylla*, *P. montezumae*, *P. patula*, *P. pseudostrobus*, *P. teocote*, *Abies religiosa*, and *Quercus* sp.—were identified in the course of interactions with technical forest service providers, producers, and government officials— through training programs and demand collection.

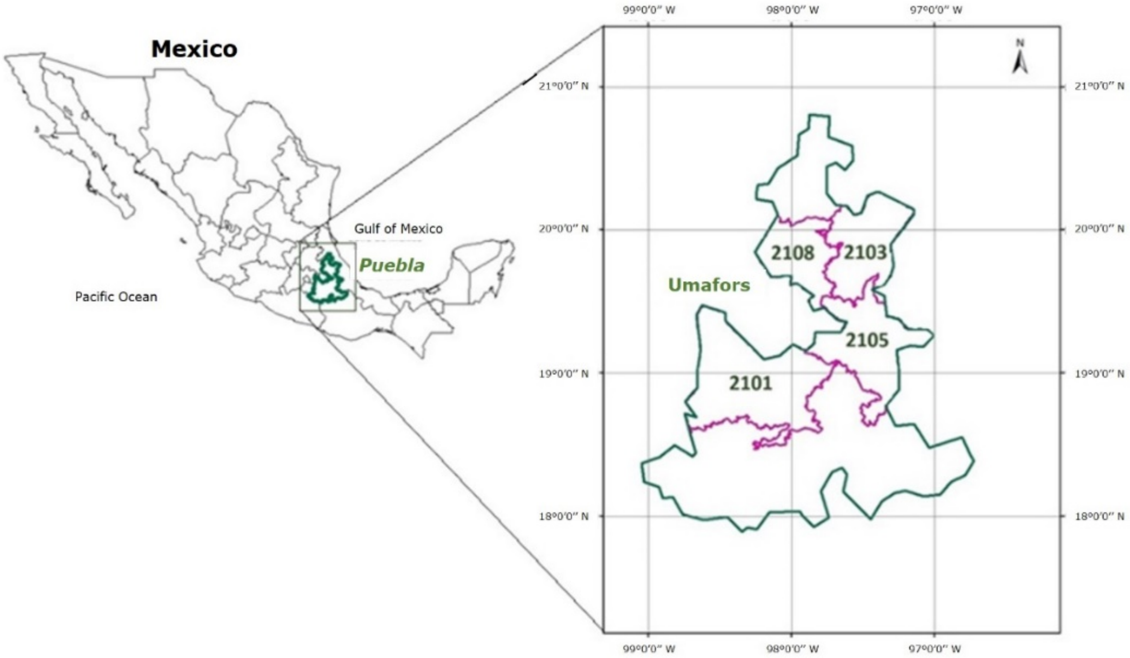

Figure 1. Location of the study area.

Data collection and measured variables

Data were collected from 1 676 random selected trees through a destructive sampling that included the felling and cutting down of the specimens. The diametric and height distributions were considered in the selection process. In order to measure and calculate the volume, each tree was divided into stem and crown. The shaft was initially cut at a height of 0.30 m above the ground level. Measurements were taken in two 0.30 m sections, one just above the stump, and the other, up to the normal diameter height (1.3 m). Subsequently, measurements of the trunk were taken at intervals of 2 m in length, until the tip of the tree was reached. The last section should not exceed 1.3 m in length. The crown was divided into two parts: (a) Branches with minor section diameters >5 cm and lengths no longer than

2 m; and (b) Branch tips and twigs with minor section diameters <5 cm. Branches and twigs with diameters under 5 cm were weighed with a model 29966 Pretul® 40 kg commercial scale and three samples were obtained (upper, middle, and lower part of the crown); its volume was estimated with a Xilometer (water drum, previously calibrated to measure volume). Measurements were made with a model 283D Forestry Suppliers® diameter tape, with a model TP30M Truper® 30 m flexometer, and the cuts were made with a model MOT-5120 Truper® 51cc 2.7 hp gasoline chainsaw.

The recorded variables were: diameter at breast height (*DBH*, cm), total tree height (*H*, m), stump height (*hs*, m), diameter outside-bark for each section (*Dob*, cm), section lengths (*l*, m), bark thickness (*Bt*, mm), outside-bark diameter for each branch section (*Dob*, cm), section length for branches with diameters above 5 cm (*Bls*, m), total crown weight (*CWT*, kg), sample weight (*Ws*, kg), and branch sample volume (*Vsbr*, cm^3).

Volume calculation

The volume of each section was calculated based on the corresponding dendrometric type (Table 1), except for the volume of the branches and twigs with a diameter of less than 5 cm, which was estimated using a weight/volume ratio and then fitted to a regression model. The total volume of the tree with bark (*Vttob*) was estimated by adding the *volume of the stump+volume of the stem+volume of the tip+volume of the crown in branches with diameters greater than 5 cm+volume of the branches and twigs with diameters less than 5 cm*. The stem volume with bark (*Vsob*) was estimated by adding *stump volume+trunk volume+tip volume*.

Table 1. Equations used in the estimation of felled trees.

Part of the tree	Dendrometric type	Formula
Tip	Apollonian paraboloid	$V = \frac{So}{2} l$
Sections and branches	Truncated apollonian paraboloid	$V = \left(\frac{So + S1}{2}\right) l$
Stump	Truncated neiloid	$V = \frac{l}{4}\left(S_0 + S_1 + \sqrt[3]{S_0 S_1}\left(\sqrt[3]{S_0 + \sqrt[3]{S_1}}\right)\right)$

V = Volume (m^3); *So* = Surface area of the smaller section (m^2); *S1* = Surface area of the larger section (m^2); S_0 and S_1 = Larger and smaller surface areas; *l* = Length (m). Source: Romahn *et al.* (1994).

Prior to the statistical analysis, outliers were reviewed and a database was created using Microsoft Office Excel (Pérez, 2006).

Adjustment of equations for total tree volume and stem volume

Berkhout's allometric equation (Prodan *et al.*, 1997) (Equation 1) was adjusted to estimate the volume of the branches and twigs under 5 cm. This volume component was determined at the genus level because of the complexity of the measurement and the minimal proportional contribution to the total tree volume. A database of 31 trees was used for the *Abies* genus, of 134 trees for *Pinus* L., and of seven trees for *Quercus.*

$$Vbt = \beta_0 DBHob^{\beta_1} \qquad (1)$$

Where:

Vbt = Volume of the branches and twigs (m^3)

β_i = Coefficients

$DBHob$ = Outside-bark diameter at breath height (cm)

The Schumacher-Hall expression (Schumacher, 1933) was used to estimate the total tree and stem volume by species (Equation 2).

$$V = \beta_0 DBHob^{\beta_1} H^{\beta_2} \quad (2)$$

Where:

V = Volume (m^3)

β_i = Parameters

$DBHob$ = Outside-bark diameter breast height (cm)

H = Total tree height (m)

The independence and homogeneous distribution of errors with zero mean and constant variance were verified: multicollinearity, autocorrelation, and heteroscedasticity. The graphical analysis of residuals *versus* estimates showed problems of heteroscedasticity in all the species; therefore, a weighted regression with a weight equal to the inverse of the variance of each observation was used. To evaluate the fit, the statistics of the Standard error of the estimator (*Ese*) and the Coefficient of determination or adjusted R^2 (R^2_{ADJ}) were estimated.

Adjustment of the taper equation

The Biging equation (Biging, 1984) was utilized to model and describe the stem profile. This expression is based on the integral form of the Bertalanffy-Richards equation with two parameters (Biging, 1984) (Equation 3).

$$d = DBH(\beta_1 + \beta_2 log\left(1 - (h/H)^{\frac{1}{m}}\left(1 - exp^{\frac{-\beta_1}{\beta_2}}\right)\right) \qquad (3)$$

Where:

d = Diameter at different heights of the stem (cm)

DBH = Diameter at breast height (cm)

β_i = Parameters to be estimated

log = Natural logarithm

h = Height for each section with respect to the ground (m)

H = Total treeheight (m)

exp = Exponential function

m = Constant value of 3 (Corral *et al.*, 1999)

Given that a correlation was detected in the observations, which breaks the principle of independence of errors, the nonlinear generalized least squares technique was applied (Huang *et al.*, 2000); the error term was expanded by a continuous autoregressive model of order x *[CAR(x)]* (Equation 4).

$$e_{ij} = \sum_{k=1}^{k=x} l_k \rho_k^{h_{ij}-h_{ij-k}} e_{ij-k} + \varepsilon_{ij} \qquad (4)$$

Where:

e_{ij} = j^{th} ordinary residue of the i^{th} tree

e_{ij-k} = j^{th} ordinary residue of the i-k^{th} tree

l_k = 1 for $j>k$ and is zero for $j \leq k$

ρ_k = Autoregressive parameter of order k to be estimated

h_{ij}-h_{ij-k} = Distance separating the j^{th}-k^{th} observation within each tree with $h_{ij}>h_{ij-k}$

ε_{ij} = Error term under independence condition

The referred error structure was fitted simultaneously with the Biging taper and Schumacher-Hall volume expressions.

In order to increase efficiency in detecting outliers in the taper, a nonparametric quadratic local adjustment (assuming a normal distribution of errors) was performed for each of the species using LOESS local regression, with a smoothing parameter of 0.3 for each species (Pompa *et al.*, 2009). The volume and taper equations were adjusted by species was carried out with the MODEL and NLIN procedure of SAS (SAS Institute Inc., 2015), which allows a dynamic update of the residuals.

Results

Table 2 shows the descriptive statistics by species for the diameter and height variables.

Table 2. Descriptive statistics of the analyzed database.

Species	Number of trees	Minimum diameter (cm)	Maximum diameter (cm)	Minimum total height (m)	Maximum total height (m)
Pinus ayacahuite C. Ehrenb. *ex* Schltdl.	141	8.3	97.4	8.58	49.3
Pinus hartwegii Lindl.	78	10.2	77	4.4	37.8
Pinus leiophylla Schiede *ex* Schltdl. & Cham.	52	11	70	7.74	34.49
Pinus montezumae Lamb.	304	2	89.3	1.54	49.25
Pinus patula Schltdl. & Cham.	230	2.1	89	7.55	40.6
Pinus pseudostrobus Lindl.	308	4.4	123	3.8	43
Pinus teocote Schltdl. & Cham.	160	8	75	5.85	40.05
Abies religiosa (Kunth) Schltdl. & Cham.	225	5.5	96	5.65	45
Quercus sp.	124	8.9	71	5.62	32.15
Overall total	1 676				

The fit of the Berkhout model for estimating the volume of branches and twigs by gender with diameter ≤ 5 cm was good; Table 3 shows the significant parameters and high values in the R^2_{ADJ}.

Table 3. Statistical parameters of the equation for estimating the volume of the branches and twigs under and equal to 5 cm by genus.

Genus	Model	β_0	β_1	Aprox. *Pr>F*	R^2_{ADJ}
Pinus L.	$Vbt = 0.000296 \times DBH^{1.7416}$	0.000296	1.7416	<0.0001	0.886
Abies Mill.	$Vbt = 0.000781 \times DBH^{1.562}$	0.000781	1.5620	<0.0001	0.946
Quercus L.	$Vbt = 0.00004 \times DBH^{2.2197}$	0.00004	2.2197	<0.0001	0.950

Vbt = Volume of the branches and twigs (m^3); *DBH* = Outside-bark diameter at breast height (cm); β_i = Estimated parameters; R^2_{ADJ} = Coefficient of determination.

The Schumacher-Hall model fit for *Vs* and *Vtt* by species showed low bias and significant parameters (Tables 4 and 5).

Table 4. Parameters and statistics of the equations for estimating the volume of the stem.

Species	**Estimated parameters**							
	β_0	β_1	B_2	$se\beta_0$	$se\beta_1$	$se\beta_2$	**Bias**	R^2_{ADJ}
Pinus ayacahuite C. Ehrenb. *ex* Schltdl.	0.00011	1.81574	0.84786	1.39E-05	0.04238	0.07566	0.01279	0.98
Pinus hartwegii Lindl.	0.00011	2.25105	0.40620	1.63E-05	0.07674	0.09383	0.04255	0.96
Pinus leiophylla Schiede *ex* Schltdl. & Cham.	0.00005	1.87382	1.03134	1.37E-05	0.06596	0.10189	-0.00145	0.98
Pinus montezumae Lamb.	0.00011	2.07621	0.60168	1.03E-05	0.03776	0.05006	-0.0011	0.98
Pinus patula Schltdl. & Cham.	0.00005	1.86843	1.00634	6.79E-06	0.03290	0.05841	0.00352	0.97
Pinus pseudostrobus Lindl.	0.00014	1.98365	0.63166	1.04E-05	0.02596	0.03924	0.02769	0.96
Pinus teocote Schltdl. & Cham.	0.00011	1.86733	0.84149	1.04E-05	0.03728	0.04905	0.00324	0.97
Abies religiosa (Kunth) Schltdl. & Cham.	0.00012	1.79730	0.87374	1.19E-05	0.03264	0.05682	0.00668	0.99
Quercus sp.	0.00017	1.92302	0.56380	1.98E-05	0.04811	0.07370	0.00429	0.97

β_i = Estimated parameters; $se\beta_i$ = Standard error of the estimator; R^2_{ADJ} = Adjusted coefficient of determination.

Table 5. Parameters and statistics of the equations for estimating the total tree volume.

Species	**Estimated parameters**							
	β_0	β_1	B_2	$se\beta_0$	$se\beta_1$	$se\beta_2$	**Bias**	R^2_{ADJ}
Pinus ayacahuite C. Ehrenb. *ex* Schltdl.	0.00018	1.88174	0.670551	2.23E-05	0.04051	0.07261	0.00996	0.98
Pinus hartwegii Lindl.	0.00017	2.30160	0.268374	2.3E-05	0.07104	0.08776	0.04185	0.96
Pinus leiophylla Schiede *ex* Schltdl. & Cham.	0.00010	1.92276	0.825071	2.19E-05	0.06696	0.09493	-0.0016	0.98
Pinus montezumae Lamb.	0.00017	2.09435	0.507474	1.43E-05	0.03960	0.05104	0.00575	0.98
Pinus patula Schltdl. & Cham.	0.00008	1.95963	0.813863	1.14E-05	0.03635	0.06156	0.00072	0.97
Pinus pseudostrobus Lindl.	0.00024	2.01848	0.464369	1.49E-05	0.02464	0.03519	0.04171	0.97
Pinus teocote Schltdl. & Cham.	0.00017	1.92822	0.672886	1.47E-05	0.03423	0.04437	0.00631	0.97
Abies religiosa (Kunth) Schltdl. & Cham.	0.00022	1.81572	0.735886	2.15E-05	0.03126	0.05476	0.00461	0.99
Quercus sp.	0.00020	2.08321	0.378606	1.99E-05	0.04313	0.06461	0.00669	0.97

β_i = Estimated parameters; seβ_i = Standard error of the estimator; R^2_{ADJ} = Adjusted coefficient of determination.

The *Vs* equations fitted for the eight timber species and one at the genus level were found to be accurate. The *Vs* model for *A. religiosa*, *P. ayacahuite*, *P. leiophylla*, and *P. montezumae* had the lowest values for standard error and bias, as well as the highest value for the Coefficient of determination.

The adjustment of *Vtt* equations show adequate coefficients of determination and standard error for the eight species and for the genus *Quercus*. The *Vtt* equation for *A. religiosa*, *P. ayacahuite*, *P. leiophylla*, and *P. montezumae* had the lowest values for standard error, bias and best fit. Table 6 shows the final equations for estimating the *Vs* and the *Vtt* by species.

Table 6. Volumetric equations determined for timber species in the temperate ecosystem of *Puebla*, Mexico.

Species	Stem volume	Total tree volume
Pinus ayacahuite C. Ehrenb. *ex* Schltdl	$Vs = 0.000115DBH^{1.815744}\ H^{0.847865}$	$Vttv0.00018DBH^{1.881747}\ H^{0.670551}$
Pinus hartwegii Lindl.	$Vs = 0.000110DBH^{1.867331}\ H^{0.841494}$	$Vtt = 0.000172DBH^{1.928221}\ H^{0.672886}$
Pinus leiophylla Schiede *ex* Schltdl. & Cham.	$Vs = 0.0000587DBH^{1.873824}\ H^{1.031340}$	$Vtt = 0.000109DBH^{1.922763}\ H^{0.825071}$
Pinus montezumae Lamb.	$Vs = 0.000117DBH^{2.076216}\ H^{0.601681}$	$Vtt = 0.000174DBH^{2.094352}\ H^{0.507474}$
Pinus patula Schltdl. & Cham.	$Vs = 0.000058DBH^{1.868438}\ H^{1.006343}$	$Vtt = 0.000088DBH^{1.959633}\ H^{0.813863}$
Pinus pseudostrobus Lindl.	$Vs = 0.000142DBH^{1.983651}\ H^{0.631666}$	$Vtt = 0.000246DBH^{2.018488}\ H^{0.464369}$
Pinus teocote Schltdl. & Cham.	$Vs = 0.000114DBH^{2.251051}\ H^{0.406208}$	$Vtt = 0.000170DBH^{2.301601}\ H^{0.268374}$
Abies religiosa (Kunth) Schltdl. & Cham.	$Vs = 0.000128DBH^{1.797302}\ H^{0.873745}$	$Vtt = 0.000221DBH^{1.815725}\ H^{0.735886}$
Quercus sp.	$Vs = 0.000172DBH^{1.923027}\ H^{0.563803}$	$Vtt = 0.000204DBH^{2.083216}\ H^{0.378606}$

Vs = Stem volume (m^3); *DBH* = Outside-bark diameter at breast height (cm); *H* = Total tree height (m); *Vtt* = Total tree volume (m^3).

Table 7 shows the parameters of the taper functions by species. The residuals and the values of the Root Mean Square Error statistics (*RMSE*) and the adjusted Coefficient of determination (R^2_{ADJ}) were reviewed.

Table 7. Parameters and fit statistics of the Biging taper equation.

Species	**Estimated parameters and adjustment statistics**					
	β_0	β_1	$se\beta_0$	$se\beta_1$	***RMSE***	R^2_{ADJ}
Pinus ayacahuite C. Ehrenb. *ex* Schltdl.	1.15806	0.347976	0.003756	0.000679	5.554092	0.9650
Pinus hartwegii Lindl.	1.20949	0.363995	0.005875	0.007064	3.663492	0.9740
Pinus leiophylla Schiede *ex* Schltdl. & Cham.	1.22164	0.431578	0.005959	0.008369	2.478267	0.9847
Pinus montezumae Lamb.	1.19754	0.343313	0.002736	0.002917	3.33656	0.9808
Pinus patula Schltdl. & Cham.	1.27765	0.493181	0.004102	0.007389	3.991821	0.9683
Pinus pseudostrobus Lindl.	1.24541	0.402689	0.003365	0.004399	3.760633	0.9711
Pinus teocote Schltdl. & Cham.	1.19086	0.326769	0.004327	0.002046	3.225456	0.9745
Abies religiosa (Kunth) Schltdl. & Cham.	1.30039	0.597817	0.004206	0.01028	4.840787	0.9674
Quercus sp.	1.24943	0.639853	0.005285	0.014327	2.729923	0.9754

β_i = Estimated parameters of the estimators; $se\beta_i$ = Standard error of the estimator; *RMSE* = Root Mean Square Error; R^2_{ADJ} = Adjusted coefficient of determination.

According to the values of the fit statistics and significance of the parameters, the equations adequately explain the stem profile for each species (Table 8).

Table 8. Stem profile equations by timber species of the temperate ecosystem of *Puebla*, Mexico.

Species	**Function of the stem profile**
Pinus ayacahuite C. Ehrenb. *ex* Schltdl.	$d = DBH(1.1580 + 0.34797\, log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp^{\frac{-1.1580}{0.34797}})\right))$
Pinus hartwegii Lindl.	$d = DBH(1.2094 + 0.36399\, log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp^{\frac{-1.3094}{0.36399}})\right))$

Pinus leiophylla Schiede *ex* Schltdl. & Cham.	$d = DBH(1.2216 + 0.43157\,log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp\tfrac{-1.2216}{0.43157})\right))$
Pinus montezumae Lamb.	$d = DBH(1.19754 + 0.34331\,log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp\tfrac{-1.19754}{0.34331})\right))$
Pinus patula Schltdl. & Cham.	$d = DBH(1.2776 + 0.49318\,log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp\tfrac{-1.2776}{0.49318})\right))$
Pinus pseudostrobus Lindl.	$d = DBH(1.2454 + 0.40268\,log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp\tfrac{-1.2454}{0.40268})\right))$
Pinus teocote Schltdl. & Cham.	$d = DBH(1.9086 + 0.32676\,log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp\tfrac{-1.9086}{0.32676})\right))$
Abies religiosa (Kunth) Schltdl. & Cham.	$d = DBH(1.3003 + 0.59781\,log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp\tfrac{-1.3003}{0.59781})\right))$
Quercus sp.	$d = DBH(1.2494 + 0.63985\,log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}(1 - exp\tfrac{-1.2494}{0.63985})\right))$

d = Diameter at a given height of the stem (cm); *DBH* = Outside-bark diameter at breast height (cm); *H* = Total tree height (m); *h* = Height determined for estimation with respect to the ground (m); *exp* = Exponential constant; *log* = logarithm.

Discussion

Equations for *Vs* and *Vtt* were fitted with the Schumacher-Hall model (Schumacher, 1933) and for stem profile with the Biging model (Biging, 1984), for eight commercial timber conifers and the *Quercus* sp. from the temperate ecosystem in *Puebla*, Mexico. A total of 27 equations were obtained; the results exhibit similarities with those obtained by Ramírez-Martínez *et al.* (2018), who fitted several models to estimate the total volume in *Pinus ayacahuite*, and concluded that the Schumacher-Hall model had the lowest bias (Bias=0.026 m^3). Tapia and Návar (2011) fitted and validated eight volume equations for *P. pseudostrobus*, and the Schumacher-Hall model proved to be accurate with a high level of reliability (R^2_{ADJ}=0.96).

Ramos-Uvilla *et al.* (2014) generated volume equations for *Pinus lawsonii* Roezl *ex* Gordon and *P. oocarpa*; the authors evaluated five models and the best fit was the Schumacher-Hall model (R^2_{ADJ}=0.99). Corral-Rivas and Návar-Cháidez (2009) evaluated seven models for estimating stem volume in *Pinus cooperi*, *P. durangensis*, *P. engelmannii*, *P. leiophylla*, and *P. herrerae* of *Durango*; again, the Schumacher-Hall model had the highest coefficients of determination, and the lowest standard errors and normally distributed errors. Rodríguez-Flores *et al.* (2019) utilized the linearized Schumacher-Hall model to estimate the volume components of nine common species or taxa groups in the temperate forests of Northwestern Mexico, and their findings show that the model fit the data adequately.

Guzmán-Santiago *et al.* (2022) reported that the Schumacher-Hall equation used in the estimation of the total tree crown for *Abies religiosa* in the state of *Puebla* was the most accurate. Vargas-Larreta *et al.* (2018) applied the Schumacher-Hall model to estimate the stem volume of wood in timber species of the states of *Chihuahua*, *Guerrero*, *Jalisco*, *Durango*, *Oaxaca*, *Michoacán*, *Puebla*, *Estado de Mexico*, *Hidalgo*, *Tlaxcala*, *Veracruz* and *Quintana Roo*, and their equations were grouped in the Forest Biometric System (Sibifor).

The Biging model (Biging, 1984) has been widely used in forest management to describe the stem profile as a function of the normal diameter. Rentería-Anima and Ramírez-Maldonado (1998) evaluated ten taper equations for *P. cooperi* in *Durango* and observed that the models showed problems in predicting the stump diameter (0.3 m). Corral *et al.* (1999) validated six taper models for *P. cooperi*, *P. durangensis*, *P. engelmannii*, *P. leiophylla* and *P. herrerae* in the forest region of *El Salto*, *Durango*, Mexico; their statistical tests showed that the Biging model (Biging, 1984) predicted the diameter profile with a good fit.

In a similar way, Pompa *et al.* (2009) compared thinning models and determined for *P. arizonica* in the Southwestern region of *Chihuahua* that the Biging model (Biging, 1984) had the greatest predictive ability and allowed the design of a compatible volume equation. Corral-Rivas and Návar-Cháidez (2009) determined that the taper function of the Biging model for *Pinus cooperi*, *P. durangensis*, *P. engelmannii*, *P. leiophylla*, and *P. herrerae* in *Durango* provides comparable volumes at the tree or stand level with conventional (Smalian, Huber or Newton) volume equations. Tamarit *et al.* (2017) generated a cubing system for the genus *Quercus* sp. in *Puebla* using the segmented model of Fang *et al.* (2000), with very good prediction.

Conclusions

The fitted equations based on the Schumacher-Hall expression at the species level provide reliable and accurate estimates for determining the stem volume and total tree volume of *Pinus ayacahuite*, *P. leiophylla*, *P. hartwegii*, *P. montezumae*, *P. patula*, *P. pseudostrobus*, *P. teocote*, *Abies religiosa*, and *Quercus* sp. in temperate forests of *Puebla*, Mexico. Equations based on the Biging model were generated to model the tapering of the same species.

Given the technical management and timber harvesting currently carried out in *Puebla*, it is considered convenient to use the equations generated to replace those developed 46 years ago by the then Forest Undersecretary of the

Secretariat of Agriculture and Hydraulic Resources. Therefore, their implementation is recommended in the elaboration and execution of timber harvesting management programs in this state.

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Conflict of interest

The authors declare that they have no conflicts of interest. José Carlos Monárrez-González and Gerónimo Quiñones Barraza declare that they did not participate in any stage of the editorial process of this article.

Contribution by author

José Carlos Monárrez-González: conceptualization of the research, data collection, statistical analysis, drafting of the manuscript; Marco Antonio Márquez-Linares: review, statistical analysis and correction of the manuscript; Juan Antonio López Hernández: in-field data collection and correction of the manuscript; Gustavo Pérez Verdín and Gerónimo Quiñones Barraza: drafting of the manuscript and statistical analysis; Xavier García Cuevas: drafting, revision and correction of the manuscript.

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